A Directed-Search Theory of Durable Goods Replacement

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Abstract

How do interest rates affect durable-goods markets in general equilibrium? I develop a directed-search model of differentiated durables in which state-dependent failure rates sort *ex ante* identical buyers and sellers across submarkets. Buyers' optimal policy partitions the state space into action and inaction regions with target quality monotonic in the holding, and market tightness is decreasing in quality. Calibrated to U.S. vehicle microdata, the model predicts that a decline in the interest rate triggers a short-run overshoot of durable expenditure; firms reallocate production toward higher-quality submarkets, and the economy converges to a stationary equilibrium of less frequent, higher-quality purchases.

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1 Introduction

Durable goods account for more than ten percent of U.S. personal consumption expenditure and are the most cyclically volatile component of household consumption (Gomes et al., 2009). Two margins shape their aggregate dynamics. Households choose how often to upgrade their durables, and sellers choose the quality composition of the goods they offer. The latter channel has been largely ignored in the literature. Its relevance has been the focus of the debate since 2020, when the semiconductor shortage shifted U.S. car-market composition toward both the premium and high-mileage tails and thinned the middle of the durability ladder—only four new cars now list under $25,000 in real terms, down from roughly a dozen a decade ago—and the distribution has not yet reverted.[1]

In general equilibrium, changes in the households' patients, that is, in discount rate affects the timing of upgrades and the quality of the goods households purchase. Once the discount rate falls, goods that survive longer become more valuable, sellers supply higher-quality goods, and the households that purchase them upgrade less frequently afterwards. This quality response works against the timing response of a model with fixed quality. If the quality response exceeds the timing response, the economy converges to a stationary equilibrium with less frequent upgrades. The discount rate affects current demand through the incentive to upgrade existing goods, and it affects future demand through the durability of the goods purchased today.

To study these questions, I build a general-equilibrium model of directed search for vertically differentiated durable goods. The market consists of a continuum of submarkets indexed by quality and price, where quality is the residual lifetime of a good. Residual lifetime depreciates deterministically, and the good fails at a Poisson rate that decreases in residual lifetime. Sellers choose a submarket, produce the good, and wait for a match. Buyers direct their search at a single submarket and trade off the arrival rate of a match against the quality of the good they target. Meetings in each submarket follow a matching technology, and free entry pins down the price schedule. The state-dependent failure rate is the key primitive. A low-quality good fails more frequently than a high-quality good, so its holder places a higher value on a short wait for an upgrade. This difference in the value of waiting sorts buyers and sellers across submarkets, and this sorting is the central

[1] Model counts and transaction prices from Cox Automotive and Kelley Blue Book, accessed via *cars.com* and *Car Edge*; see also C. Winston, "The Affordable Car Is Dead. What Happened?" *The New York Times*, April 13, 2026.

mechanism of the paper.

I establish three theoretical results. First, the optimal upgrade policy partitions the state space into an *action region*, where buyers search with a target quality that increases in the current holding, and an *inaction region*. Unlike previous models in the literature where lumpiness arises from fixed costs, here it does from the interaction of failure risk and search frictions. Second, market tightness decreases in the buyer's current quality. Sellers enter in greater numbers in the submarkets targeted by buyers close to failure, accept a lower meeting rate there, and receive a higher price in return. This is the supply-side counterpart of buyer sorting. Third, a lower discount rate raises the threshold at which buyers start searching and raises the target quality at any given market tightness, because the cost sellers bear while financing production before a sale falls and buyers value durability more. The change in the upgrade rate of the goods purchased then splits into a timing response, which raises upgrades at fixed quality, and a quality response, which lowers them. A single change in the discount rate reallocates demand and supply across submarkets at once, and relative to (S, s) models the directed-search environment makes this compositional response of supply a general equilibrium object.

I calibrate the model to U.S. vehicle data from the Panel Study of Income Dynamics (PSID), where each vehicle's expected residual lifetime (ERL) comes from its age, its manufacturer, and the survival schedules of automobiles and light trucks. The targets are the ERL distribution among owners, the mean ERL at purchase, mean vehicle age, the annual adjustment rate, the fraction with zero vehicles, price dispersion, and log-price skewness.

The quantitative exercise is a permanent decline in the discount rate. I compare the stationary equilibria before and after it, and between the two I compute a transition in which buyers switch at once to the new stationary target policy while sellers, who start with the old stationary stock, adjust gradually. Durable expenditure initially overshoots, and the new stationary equilibrium buyers hold goods of higher quality and upgrade them 1.4 percent less often. The age of their holdings falls by 0.02 years, whereas a model with fixed quality predicts 9 percent more upgrades and a stock of goods that is 0.26 years younger. Section 5 splits the change in the upgrade rate into a timing response at fixed quality and a quality response, and Section 6 measures both.

This paper relates to three strands of literature. The first is the literature on durable-goods consumption, which studies durable adjustment within the fixed-cost (S, s) framework pioneered

by Grossman and Laroque (1990) and extended by Bernanke (1985), Bénabou (1989), Leahy and Zeira (2005), Caplin and Leahy (2006), Berger and Vavra (2015), McKay and Wieland (2021), Attanasio et al. (2022), and Bertolotti et al. (2023). Sellers' quality choices are independent of households' upgrade decisions in the standard fixed-cost specification. Households adjust upgrade timing within a given quality composition of supply. My contribution is to develop a general-equilibrium framework in which lumpy upgrades emerge from the interaction of search frictions and state-dependent failure rates, and in which the distribution of sellers across qualities responds to aggregate shocks. A closely related paper is Gavazza and Lanteri (2021), which studies credit and quality differentiation in durable-goods markets. I treat quality as a continuum evolving through depreciation and failure, model supply through directed search, and study stationary comparative statics in the discount rate. A complementary strand studies the producer side of investment-goods markets. For example, Bertolotti et al. (2024) characterize oligopolistic pricing, dynamic state-dependent markups, and capacity constraints faced by upstream producers. My analysis focuses on the equilibrium quality composition of goods offered to households.

The second strand is the directed-search literature.[2] Inderst and Müller (2002) study durable goods under competitive search with asymmetric information in a static environment, while Gomis-Porqueras et al. (2017) examine sellers' costly advertising. My contribution to this literature is twofold. I develop a dynamic framework with depreciation and state-dependent failure that is portable to other durable-asset markets, and I introduce a sorting mechanism grounded in failure rates, where the existing literature assumes match supermodularity.

The third strand is the literature on search-based price dispersion (Butters, 1977, Varian, 1980, Burdett and Judd, 1983a, Bénabou, 1989, Galenianos et al., 2012, Kaplan et al., 2019, Nord, 2022), and the empirical work of Kaplan and Menzio (2015). I extend this line of research from nondurables to durables, and from search over prices to joint search over prices and qualities.

[2]For surveys, see Moen (1997), Acemoglu and Shimer (2000), Shi (2009), Menzio and Shi (2011), Albrecht et al. (2016), Wright et al. (2021).

2 Model

2.1 Environment

Time is continuous and the horizon infinite. A unit measure of ex ante identical buyers faces a freely entering pool of ex ante identical sellers, and everyone discounts the future at the common rate $\rho > 0$.[3] Each buyer holds at most one indivisible durable good with residual lifetime $x \in [0, \bar{x}]$, where $\bar{x} < \infty$. The good depreciates deterministically at rate $\delta > 0$, $\dot{x} = -\delta x$, and fails at the state-dependent Poisson rate $\lambda(x)$.[4] A good that fails jumps to $x = 0$. The failure rate $\lambda : (0, \bar{x}] \to \mathbb{R}_+$ is continuous and strictly decreasing on $(0, \bar{x}]$, with $\lambda(\bar{x}) = 0$ and $\lim_{x \downarrow 0} \lambda(x) = \infty$, so older goods fail more frequently. A buyer in the non-owner state $x = 0$ faces zero failure risk. An owner with $x > 0$ receives flow utility $d > 0$ and a non-owner receives zero. Firms produce a new good with initial residual lifetime $\zeta \in [0, \bar{x}]$ at cost $c(\zeta)$, with $c(0) = 0$, $c' > 0$, $c'' > 0$, and $c(\bar{x}) < d/\rho$, and operating sellers pay a flow cost $k > 0$ until they sell. At a match the buyer acquires residual lifetime ζ and scraps the current good if $x > 0$. Let τ_ζ denote the failure time of a good that starts at residual lifetime ζ and is held until failure. Its survival probability is $\Pr(\tau_\zeta > t) = \exp\{-\int_0^t \lambda(\zeta e^{-\delta s})\, ds\}$. I impose that for some quality the production cost lies below the discounted flow utility from the good held until failure, $c(\zeta) < d\, \mathbb{E} \int_0^{\tau_\zeta} e^{-\rho t}\, dt$ for some $\zeta \in (0, \bar{x}]$.

2.2 Market structure

Buyers and sellers trade in a continuum of submarkets $\psi = (\zeta, p) \in [0, \bar{x}] \times \mathbb{R}_+$. Sellers produce for a single submarket and post its price. A buyer directs search at one submarket. I denote the mass of sellers in a submarket by s and the mass of searching buyers by b. Market tightness is $\theta(\psi) = s/b$. Meetings follow a constant-returns technology. A buyer meets a seller at Poisson rate $r(\theta)$, where r is bounded, strictly increasing, and strictly concave. A seller meets a buyer at rate

$$q(\theta) = \frac{r(\theta)}{\theta}.$$

[3] Buyers can equivalently hold a numéraire good with constant marginal utility normalized to one and invest it at the safe rate ρ.

[4] A good becomes unusable after an engine failure, obsolescence, or an uneconomic repair.

2.3 Discussion

Flow utility. Owners receive the constant flow utility d for a longer expected period at higher quality because $\lambda(x)$ falls. Buyers in Gowrisankaran and Rysman (2012) and Hendel and Nevo (2013) face idiosyncratic taste shocks. Berger and Vavra (2015) model utility from the stock. I add horizontal differentiation in Appendix C.

Search protocol. Directed search has three features that matter here. It delivers posted terms of trade, it nests general equilibrium, and, under free entry, the stationary equilibrium is block recursive (Menzio and Shi, 2011), so policies are independent of the distribution— a property I exploit in the theoretical analysis. Random search in the tradition of Burdett and Judd (1983a) does not allow for heterogeneity in both prices and qualities, while bargaining-based random-matching models require auxiliary protocols that this paper does not need.

3 Equilibrium

3.1 Equilibrium conditions

I denote the value of holding a good of residual lifetime x by $V(x)$ and the value of non-ownership by $U \equiv V(0)$. The buyer's expected gain from search is $D(x)$. A seller who has produced quality ζ and posted price p has value $J(\zeta, p)$.

Buyers. The buyer's Hamilton–Jacobi–Bellman (HJB) equation is

$$\rho\, V(x) \;=\; d \;-\; \delta\, x\, V'(x) \;-\; \lambda(x)\big(V(x) - U\big) \;+\; D(x), \tag{1}$$

with search gain

$$D(x) \;=\; \max_{\zeta,\theta}\; r(\theta)\big(V(\zeta) - p(\zeta,\theta) - V(x)\big), \tag{2}$$

so that $V(0) = U$ satisfies $\rho U = D(0)$, because a non-owner searches, receives zero flow utility, and faces zero drift and zero failure risk. The term $-\lambda(x)(V(x) - U)$ in (1) records the return to the non-owner's continuation value $V(0) = U$ after failure.

Sellers. A seller who has produced quality ζ and entered the corresponding submarket earns value

$$\rho\, J(\zeta, p) \;=\; -k \;+\; q(\theta)\big(p - J(\zeta, p)\big). \tag{3}$$

Free entry requires $c(\zeta) \geq J(\zeta, p)$ and $\theta(\zeta, p) \geq 0$ with complementary slackness. I solve the free-entry condition for p to obtain

$$p \;=\; \frac{(\rho + q(\theta))\, c(\zeta) + k}{q(\theta)}. \tag{4}$$

3.2 Existence and uniqueness

The buyer meeting rate satisfies $r(0) = 0$ and $r'(0+) = \infty$. The function r is continuously differentiable on $(0, \infty)$. I write $\bar{r} = \sup_{\theta \geq 0} r(\theta) < \infty$.

Definition 1. *A* recursive stationary equilibrium *is a tightness function $\theta(\cdot)$, a buyer value function V, a search value D, buyer policies (ζ^*, θ^*), a seller value J, a price schedule p, and a distribution G such that (i) (ζ^*, θ^*) solves (2) given (V, p), (ii) V satisfies (1), (iii) free entry holds, and (iv) G is stationary.*

A seller pays the production cost before entering a submarket and pays the operating cost k until a sale. Free entry equates the value of entering with the production cost and thereby determines the posted price. The price covers production and the expected operating cost during the wait for a buyer. Sellers of the same quality in submarkets with different market tightness post different prices. In the tighter submarket buyers meet sellers faster and sellers wait longer for a sale. The price in that submarket carries the cost of the longer wait.

A buyer's choice of submarket depends on the continuation value $V(\zeta)$ at the target quality. Discounting bounds the weight of that continuation value in the current value. The free-entry price in (4) depends on quality and market tightness alone. The stationary equilibrium is therefore block recursive in the sense of Shi (2009) and Menzio and Shi (2011). I solve for the buyer value and the search choices at the free-entry prices first. The holdings distribution follows from these choices. The seller stock in each submarket equals the mass of buyers searching there times the market tightness they chose.

Theorem 1. *A stationary recursive equilibrium exists. Its bounded buyer value V is unique, continuous on $[0, \bar{x}]$, and continuously differentiable at positive qualities, with a left derivative at $\bar{x}$. Each stationary optimal search policy has a unique stationary holdings distribution. A unique active maximizer in (6) pins down a unique stationary allocation and active prices.*

Proof. See Appendix A.1. ■

4 Equilibrium characterization

I denote the density of positive holdings by $g(x)$. Figure 1 plots the equilibrium at the calibrated parameters of Section 6.

4.1 Market tightness

The buyer value function V is strictly increasing on $[0, \bar{x}]$ (Lemma A.1 in Appendix A.1). Buyers and sellers share the joint surplus $S(x, \zeta) \equiv V(\zeta) - c(\zeta) - V(x)$ from a purchase.

A buyer choosing a given target quality pays for a faster match through the price. Additional sellers raise market tightness and shorten the buyer's expected wait, but the seller meeting rate $q(\theta)$ falls. Since the sellers have already paid the production cost to enter the submarket, the free entry requires a higher price to compensate them for the longer expected wait. A buyer with a larger surplus gains more from receiving the upgrade sooner and accepts this price increase. Higher current quality raises the value of keeping the current good and lowers the gain from completing a given upgrade.

Lemma 1. *For a fixed target ζ, the optimal market tightness is*

$$\theta^*(x, \zeta) = \begin{cases} r'^{-1}\left(\dfrac{\rho c(\zeta) + k}{S(x, \zeta)}\right), & S(x, \zeta) > 0, \\ 0, & S(x, \zeta) \leq 0. \end{cases} \tag{5}$$

Positive market tightness increases strictly with the surplus at that target. Every optimal offer satisfies $\theta^* \leq \bar{r}d/(\rho k)$. Optimal market tightness is weakly decreasing in current quality. The search gain D is continuous and weakly decreasing, with strict decrease over states where $D > 0$.

Proof. Substitution of the free-entry price gives

$$D(x) = \max_{\zeta,\theta} \{r(\theta)S(x,\zeta) - [\rho c(\zeta) + k]\theta\}. \tag{6}$$

For a positive fixed surplus, strict concavity of r makes the objective strictly concave in θ. Its derivative tends to infinity at zero and to $-[\rho c(\zeta)+k] < 0$ at infinity. Its unique maximizer satisfies (5). A nonpositive surplus leaves market tightness at zero, and strict decrease of r' delivers the surplus comparison. The proof of Theorem 1 gives $0 \leq V \leq d/\rho$. Hence the objective is at most $\bar{r}d/\rho - k\theta$. The choice $\theta = 0$ earns zero, so every optimum satisfies the stated bound. The maximum is continuous on this compact choice set. For $x_1 < x_2$, every fixed offer earns weakly less at x_2. If an optimal offer at x_2 earns a positive gain, evaluating it at x_1 gives

$$D(x_1) \geq D(x_2) + r(\theta^*(x_2))[V(x_2) - V(x_1)] > D(x_2).$$

Let offer i be optimal at x_i and write $r_i = r(\theta_i)$. Its part independent of current quality is

$$B_i = r_i[V(\zeta_i) - c(\zeta_i)] - [\rho c(\zeta_i) + k]\theta_i.$$

The two optimality inequalities are $B_1 - r_1 V(x_1) \geq B_2 - r_2 V(x_1)$ and $B_2 - r_2 V(x_2) \geq B_1 - r_1 V(x_2)$. Adding gives

$$(r_1 - r_2)[V(x_2) - V(x_1)] \geq 0.$$

Strict increase of V and r implies $\theta_1 \geq \theta_2$. ■

Panel (c) of Figure 1 plots $\theta^*(x)$.

4.2 Optimal upgrade

A non-owner receives zero flow utility until a purchase. Any purchase at a price below the value of ownership is therefore worth searching for. An owner already receives the flow d from the current good, but an upgrade allows to extend the expected period of ownership. The length of that extension depends on the current good. If a good has a high failure rate, its owner is impatient to replace, therefore is willing to accept a high meeting rate for a lower quality good. Instead, a

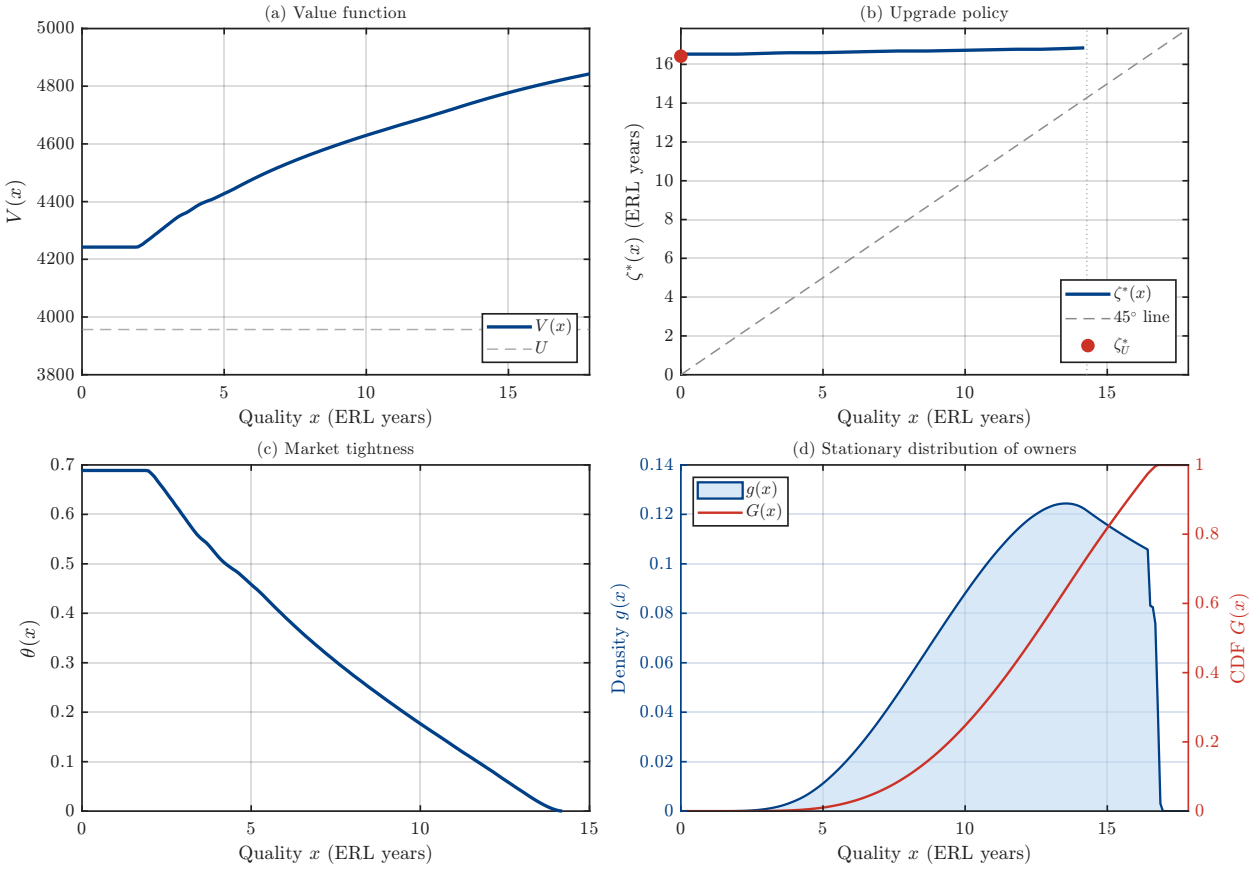


Figure 1. Calibrated equilibrium objects. Panels (a)–(d) show the value function $V(x)$, upgrade policy $\zeta^*(x)$ with the 45° line, market tightness $\theta(x)$ (Lemma 1), and stationary density $g(x)$ (Theorem 2), respectively. The red dot marks $\zeta^*(0)$. Horizontal axes show years of expected residual lifetime (ERL), or σx, where $\sigma\bar{x} = 17.9$ years is the ERL of a new vehicle of the most durable make. Policies use the calibration of Section 6.

good near $\bar{x}$ already has the longest expected life available in the market and a replacement adds little. Production cost is positive and the surplus from upgrading at the top of the quality range is negative. Depreciation lowers residual lifetime over time and moves the owner from inaction into search at the threshold x_c of Lemma 2.

Lemma 2. *A unique threshold $x_c \in (0, \bar{x})$ satisfies*

$$V(x_c) = \max_{\zeta \in [0,\bar{x}]} \{V(\zeta) - c(\zeta)\}.$$

Buyers choose positive market tightness on $[0, x_c)$ and zero market tightness on $[x_c, \bar{x}]$. Every active target lies in $(x, \bar{x}]$.

Proof. Suppose $D(0) = 0$. Then $U = 0$ by the non-owner HJB. A buyer who holds until failure a good whose production cost lies below its discounted flow utility obtains more than its production cost. Its surplus at zero is positive. The condition $r'(0+) = \infty$ then produces a positive search gain, a contradiction. Thus $D(0) > 0$ and $\max_\zeta\{V(\zeta) - c(\zeta)\} > U$. The maximum surplus $\max_\zeta\{V(\zeta) - c(\zeta)\} - V(x)$ is continuous and strictly decreasing in x. At $\bar{x}$, every target has strictly negative surplus. A lower target has lower value, and target $\bar{x}$ has positive production cost. Compactness makes the maximum strictly negative. There is a unique zero $x_c \in (0, \bar{x})$. A positive surplus gives positive search gain at sufficiently small market tightness because $r'(0+) = \infty$. A nonpositive maximum surplus leaves the search gain and the optimal market tightness at zero. A target $\zeta \leq x$ has nonpositive surplus. Every active target therefore exceeds current quality. ■

An active buyer values an increase in target quality through the longer expected period of durable services after purchase. The seller pays the additional production cost before the match and recovers it at the eventual sale, at a fixed meeting rate and flow operating cost for a given market tightness, so discounting erodes the extra revenue and free entry requires a price increase that exceeds the additional production cost. The buyer receives the quality improvement and pays this price increase at the match. At the chosen target, the buyer's marginal value of quality equals the price increase per unit of quality.

Lemma 3. *If $\theta^*(x) > 0$, then (i) $V'(\zeta^*(x)) > 0$ and (ii) $V'(\zeta^*(x)) > c'(\zeta^*(x))$. At $\bar{x}$, the derivative of V is its left derivative.*

Proof. At an interior target, the first-order condition for ζ in (6) is

$$q(\theta)\big(V'(\zeta) - c'(\zeta)\big) \;=\; \rho\, c'(\zeta). \tag{7}$$

The right-hand side is strictly positive because $c' > 0$. At target $\bar{x}$, the first-order inequality is $q(\theta)[V'_{-}(\bar{x}) - c'(\bar{x})] \geq \rho c'(\bar{x}) > 0$. Both cases give the stated inequalities. ■

A buyer with higher current quality faces a lower failure rate while searching, selects lower market tightness, and accepts a longer expected wait for an upgrade. In that submarket fewer sellers compete for each searching buyer, so sellers meet buyers faster, recover their production expenditure sooner, and require a smaller payment for delaying the recovery of any additional production cost. Evaluated at the target chosen by a buyer with lower current quality, a further quality increase therefore gains more than its additional price under this lower market tightness, and the buyer raises target quality. The target comparison uses the production cost and the seller meeting rate at each chosen offer. At a fixed positive market tightness, target quality maximizes

$$V(\zeta) - \left[1 + \frac{\rho}{q(\theta)}\right] c(\zeta). \tag{8}$$

An interior maximum equates marginal value with marginal price. A unique interior maximum also yields a strict target comparison across buyers.

Lemma 4. *Suppose* (8) *has a unique interior maximizer at every market tightness selected by active buyers. Then optimal market tightness is strictly decreasing and target quality is strictly increasing over the action region.*

Proof. For $x_1 < x_2$ in the action region, Lemma 1 gives $\theta_1 \geq \theta_2 > 0$. Equal market tightness gives the same unique target. The first-order condition for market tightness then requires

$$r'(\theta)[V(\zeta) - c(\zeta) - V(x_i)] = \rho c(\zeta) + k \qquad (i = 1, 2),$$

which contradicts strict increase of V. Thus $\theta_1 > \theta_2$. Strict concavity of r, together with $r(0) = 0$, makes $q(\theta) = r(\theta)/\theta$ strictly decreasing. Write $a_i = 1 + \rho/q(\theta_i)$, so $a_1 > a_2$. Optimality of the two

targets in (8) gives

$$(a_1 - a_2)[c(\zeta_2) - c(\zeta_1)] \geq 0.$$

Strict increase of c implies $\zeta_2 \geq \zeta_1$. Equal interior targets in the first-order condition

$$\frac{V'(\zeta)}{c'(\zeta)} = 1 + \frac{\rho}{q(\theta)} \tag{9}$$

require equal market tightnesses. Hence $\zeta_2 > \zeta_1$. ■

Strict concavity of (8), together with an interior maximum, is sufficient for the target condition in Lemma 4. This condition applies to the target objective at the selected market tightnesses. Purchases are lumpy. A purchase raises residual lifetime at a single date, whereas depreciation reduces it continuously between purchases. In the inaction region buyers receive durable services and keep their goods, and they start searching once their goods enter the action region. Because expenditure occurs at the eventual match, the search decision and the purchase fall on different dates. Buyers also choose the quality of the next purchase, and their target varies with the current holding because the failure rate determines how they value the wait for an upgrade. Buyers close to failure search for a quicker upgrade at lower quality. Buyers with higher current quality accept longer expected waits for higher quality.

Proposition 1. *Under the target condition of Lemma 4, the optimal policy partitions the full state space into (i) an* action region $[0, x_c)$*, on which* $\zeta^*(x)$ *is strictly increasing and strictly above the* 45° *line, and (ii) an* inaction region $[x_c, \bar{x}]$*, on which* $\zeta^*(x) = 0$*.*

Proof. Immediate from Lemmas 2 and 4. ■

Panel (b) of Figure 1 plots $\zeta^*(x)$ above the 45° line on $[0, x_c)$ and at zero from x_c onward. The red dot marks $\zeta^*(0)$, which lies in the inaction region in the calibrated figure. Buyers are identical before they purchase goods, and their holdings subsequently differ because failure and successful search occur at different dates. Holding different goods, they direct search toward different qualities according to the failure rate of the good they hold, and sellers enter the submarkets that they choose. An active seller recovers the production cost and the cost of waiting through the posted price, so the free-entry price varies with quality through the production cost and with market tightness through

the expected wait before sale. The distribution of transaction prices weights each posted price by the transactions in that submarket.

Corollary 1. *Buyers trade a continuum of target qualities in the stationary equilibrium, each at the free-entry price of its submarket.*

Proof. By Theorem 2 the stationary distribution has a density on positive qualities. A non-owner purchases at rate $r(\theta^*(0)) > 0$ and acquires quality $\zeta^*(0) > 0$, and depreciation carries these goods through every quality below $\zeta^*(0)$, so the density is positive almost everywhere on $(0, \zeta^*(0))$. Buyers on $(0, \min\{x_c, \zeta^*(0)\})$ search at a positive rate, and on this interval ζ^* is strictly increasing by Proposition 1. A set of buyers with positive mass therefore trades a continuum of target qualities, and free entry assigns to each chosen offer the price (4). ■

4.3 Stationary distribution

Three flows determine the distribution over positive qualities. Depreciation moves every owner downward. Purchases move households from their current quality to their target quality and place the inflow at the qualities the targets select. Search removes households from the action region. Every good is held at or below the quality at which it was purchased and the distribution has its support below the highest target. I write $m_0(t) = G_t(\{0\})$ for the mass at zero and $R(x) = r(\theta^*(x))$ for the buyer meeting rate under the selected stationary policy. The largest chosen target is $\zeta_{\max} = \sup_{0 \le x < x_c} \zeta^*(x)$. For a set of positive qualities A, the purchase inflow is

$$\int_{[0,\bar{x}]} \mathbf{1}_{\{\zeta^*(y) \in A\}} R(y)\, G_t(dy). \tag{10}$$

This expression includes the purchases from zero.

Theorem 2. *Each stationary optimal search policy has a unique nondegenerate stationary holdings distribution G. It has a mass $m_0 \in (0,1)$ at zero and a density on positive qualities. Its support is contained in $[0, \zeta_{\max}]$.*

Suppose the positive part of G_t has density $g(\cdot, t)$ and the active target policy is continuously differentiable with $\zeta^{*\prime} > 0$. Let $y = \zeta^{*-1}(x)$ on $\zeta^*((0, x_c))$, and let δ_z denote a unit point mass at z.

The purchase inflow and outflow are

$$\Phi_{\text{in}}(x,t) = \frac{g(y,t)R(y)}{\zeta^{*\prime}(y)}\mathbf{1}_{\{x\in\zeta^*((0,x_c))\}} + m_0(t)R(0)\delta_{\zeta^*(0)},$$
$$\Phi_{\text{out}}(x,t) = R(x)g(x,t).$$

The density satisfies the flow balance

$$\partial_t g(x,t) = \delta\,\partial_x[xg(x,t)] - \lambda(x)g(x,t) + \Phi_{\text{in}}(x,t) - \Phi_{\text{out}}(x,t) \tag{11}$$

in the sense of distributions. The mass at zero satisfies

$$\dot{m}_0(t) = \int_{(0,\bar{x}]} \lambda(x)g(x,t)\,dx - R(0)m_0(t). \tag{12}$$

The stationary right-hand sides are zero, and $m_0 + \int_0^{\bar{x}} g(x)\,dx = 1$. *Equation* (10) *gives the purchase inflow for general stationary target policies.*

Proof. Appendix A.1 gives a common positive lower bound on the probability of reaching zero during a fixed period under any stationary policy. Its contraction argument delivers a unique invariant distribution and strictly positive mass at zero. Lemma 2 gives $R(0) > 0$ and $\zeta^*(0) > 0$. A buyer at zero waits a finite expected time $1/R(0)$ before purchase and then spends a positive expected time in ownership. The common return bound makes the expected time back to zero finite. Hence $m_0 \in (0,1)$. During each ownership spell, depreciation makes positive qualities move occupying $1/(\delta x)$. The expected number of purchases in a return cycle is finite because their rate is bounded. The normalized expected occupation measure therefore has a density on positive qualities. Purchases enter at qualities at most $\zeta_{\max}$, and depreciation lowers quality. This gives the support bound. Depreciation supplies the flux $\delta\partial_x(xg)$, and failure removes the flow λg. Purchases remove $R(x)g(x,t)$ from source quality x. Owner purchases enter their targets through (10). Changing variables from y to $\zeta^*(y)$ produces the Jacobian term in Φ_{in}. Purchases from zero enter separately at $\zeta^*(0)$. Failure is the inflow to zero, and purchases are its outflow. These balances are (11) and (12). ■

5 Comparative statics

In this section, I compare stationary equilibria across discount rates. A lower discount rate raises buyers' valuation of future durable services and lowers the price at which sellers are willing to produce and wait for a buyer to show up.

Lemma 5. *Let $\rho_1 > \rho_2 > 0$, hold the other primitives fixed, and let $c(\zeta) < d\,\mathbb{E}\int_0^{\tau_\zeta} e^{-\rho_1 t}\,dt$ for some $\zeta \in (0, \bar{x}]$. Then $V_2(x) > V_1(x)$ for every $x \in [0, \bar{x}]$, and $x_c(\rho_2) > x_c(\rho_1)$ whenever $V_2 - V_1$ is strictly increasing.*

Proof. See Appendix A.2. ■

A more patient buyer values every good more and, when the increment in value is larger for better goods, starts searching at a higher current quality. This is the demand side of the comparison. The supply side works through the price. At fixed target quality and market tightness the free-entry price falls by $(\rho_1 - \rho_2)c(\zeta)/q(\theta)$, and the price increase per unit of quality in (8), $c'(\zeta)[1 + \rho/q(\theta)]$, falls with it. The next lemma isolates what the lower price of quality does to buyers' targets. I write $\zeta(\theta; \rho)$ for the maximizer of (8) at market tightness θ and discount rate ρ, so that $\zeta_i(x) = \zeta(\theta_i(x); \rho_i)$ at every active state.

Lemma 6. *Let $\rho_1 > \rho_2 > 0$, hold the other primitives fixed, and suppose (8) has a unique interior maximizer at every positive market tightness at both discount rates.*

(i) If $V_2 - V_1$ is strictly increasing, then $\zeta(\theta; \rho_2) > \zeta(\theta; \rho_1)$ at every $\theta > 0$.

(ii) At a fixed discount rate, $\zeta(\theta; \rho)$ is strictly decreasing in θ.

Proof. See Appendix A.2. ■

Part (i) adds the higher valuation of Lemma 5. Both parts hold market tightness fixed, and in equilibrium market tightness moves too. Part (ii) says in which direction this matters, since a buyer who searches at higher tightness targets lower quality. The change in target quality at an active state therefore splits into

$$\zeta_2(x) - \zeta_1(x) = \underbrace{\zeta(\theta_1(x); \rho_2) - \zeta(\theta_1(x); \rho_1)}_{\text{fixed-tightness response}} + \underbrace{\zeta(\theta_2(x); \rho_2) - \zeta(\theta_1(x); \rho_2)}_{\text{tightness response}}, \tag{13}$$

where the fixed-tightness response is strictly positive by part (i) and the tightness response is weakly negative by part (ii) if $\theta_2(x) \geq \theta_1(x)$. Purchased quality rises when the first term is the larger of the two. The last step turns these changes into the rate at which the purchased goods are upgraded. The period of ownership of a good ends at its failure or at the purchase of an upgrade. Under a tightness policy $\theta(\cdot)$, a good purchased at quality ζ has quality $\zeta e^{-\delta a}$ at age a and leaves ownership at rate $\lambda(\zeta e^{-\delta a}) + r(\theta(\zeta e^{-\delta a}))$. In the stationary equilibrium purchases at each quality equal the exits of goods of that quality, so purchases per unit of time per unit of stock of goods purchased at quality ζ equal the inverse of the expected period of ownership,

$$n(\zeta, \theta) = \left[\int_0^\infty \exp\left\{ - \int_0^a \left[\lambda(\zeta e^{-\delta s}) + r(\theta(\zeta e^{-\delta s})) \right] ds \right\} da \right]^{-1}, \tag{14}$$

the upgrade rate. A lower discount rate moves the upgrade rate through the two margins of the previous steps, the tightness at which the good's owner searches and the quality of the good itself.

Proposition 2. *Let $\rho_1 > \rho_2 > 0$, hold the other primitives fixed, and let $c(\zeta) < d\,\mathbb{E}\int_0^{\tau_\zeta} e^{-\rho_1 t}\, dt$ for some $\zeta \in (0, \bar{x}]$. For every x with $\zeta_1(x) > 0$ and $\zeta_2(x) > 0$, the change in the upgrade rate of the goods purchased by a buyer at state x is*

$$n(\zeta_2(x), \theta_2) - n(\zeta_1(x), \theta_1) = \underbrace{n(\zeta_1(x), \theta_2) - n(\zeta_1(x), \theta_1)}_{\textit{timing response}} + \underbrace{n(\zeta_2(x), \theta_2) - n(\zeta_1(x), \theta_2)}_{\textit{quality response}}, \tag{15}$$

where the timing response is weakly positive if $\theta_2 \geq \theta_1$ on $[0, \bar{x}]$ and the quality response is strictly negative if $\zeta_2(x) > \zeta_1(x)$.

Proof. See Appendix A.2. ■

The timing response is what a model with a fixed set of qualities predicts. The action region expands and market tightness rises, so buyers upgrade earlier. The quality response is what the supply of quality adds. A good purchased at higher quality fails at a lower rate, and its owner searches at lower market tightness by Lemma 1, so it stays in ownership longer. The sign of the sum depends on how large the quality response is relative to the timing response, and that is a quantitative question. Figure 2 compares $\beta = 0.92$ and $\beta = 0.96$ at the calibrated parameters. Target quality is higher over a wider action region in panel (b), so the fixed-tightness response in

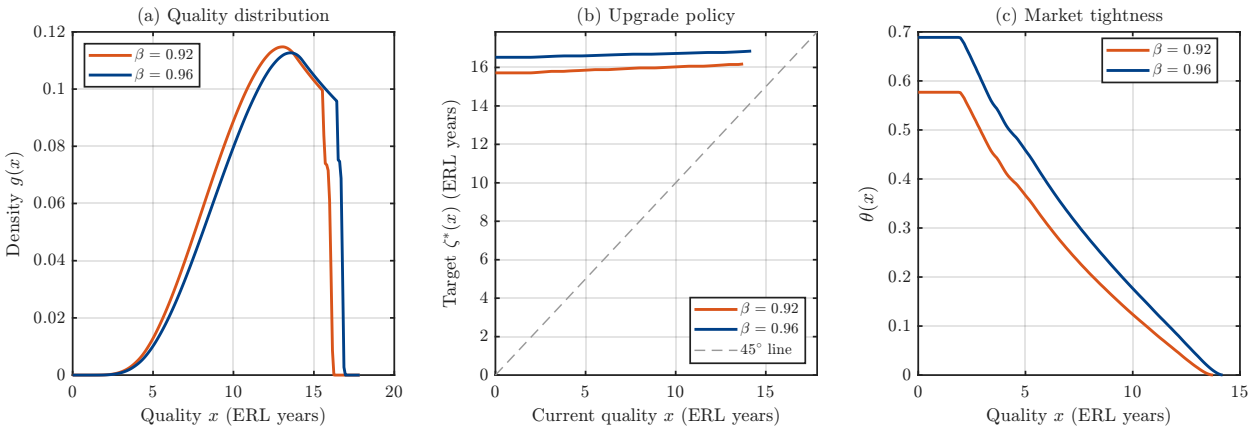


Figure 2. Comparative statics in the discount rate. $\beta = 0.92$ (orange) versus $\beta = 0.96$ (blue, baseline). The panels show the stationary density $g(x)$ in (a), upgrade policy $\zeta^*(x)$ in (b), and tightness $\theta(x)$ in (c). Parameter values are the calibrated values of Section 6.

(13) exceeds the tightness response at every active state, and market tightness is higher at every state in panel (c).

6 Quantitative analysis

I calibrate the model to the U.S. vehicle market and then study a permanent decline in the discount rate, after which buyers change their target quality and initially meet sellers faster on pre-produced inventory.

6.1 Calibration

Buyers meet sellers according to a bounded matching function with decreasing returns in market tightness, sellers pay production cost $c(\zeta) = c\,\zeta^{\mu}$, and owners receive constant flow utility d (Appendix E). The annual discount rate is $\rho = -\ln(0.96)$, the cost scale is normalized to $c = 0.01$, and matching congestion is fixed at $B = 2$. The failure rate $\lambda(x)$ comes from the survival and scrappage schedules for automobiles and light trucks of model year 1990 in Davis (2000), combined with the durability of new vehicles in the PSID (Appendix D). The top quality $\bar{x}$ is the ERL of a new vehicle of the most durable make, 17.9 years, with $\sigma = 17.9/\bar{x}$ being state units into years.

The remaining six parameters $(\delta, A, d, k, \mu, \gamma)$ in Table 1 come from simulated method of moments (SMM) on ten targets, namely the mean, standard deviation, P_{25} and P_{75} of ERL among owners, the mean ERL at purchase, mean vehicle age, the annual adjustment rate, the fraction with

zero vehicles, price dispersion, and log-price skewness (Appendix D).

The level and dispersion of ERL among owners discipline δ, since residual lifetime falls at rate δx between purchases. The annual adjustment rate and mean vehicle age discipline A. The fraction with zero vehicles is informative about d relative to k. Mean ERL at purchase disciplines μ, because the gap between the mean purchased quality and $\bar{x}$ measures how fast production cost rises with quality. Price dispersion disciplines k, and log-price skewness is primarily informative about γ.

Parameter	Symbol	Value	Std. error	Source
Externally calibrated				
Discount rate	ρ	0.0408	—	Annual discount factor = 0.96
Cost scale	c	0.01	—	Normalization
Matching congestion	B	2.0	—	Fixed
Failure rate	$\lambda(x)$	Schedule	—	ORNL Tables 6.9–6.10, model year 1990
ERL of a new vehicle	$\sigma\bar{x}$	17.85	—	ORNL Tables 6.9–6.10
Internally calibrated (Internally calibrated)				
Depreciation rate	δ	0.07018	(0.00039)	Internally calibrated
Matching efficiency	A	0.7388	(0.029)	Internally calibrated
Flow utility	d	223.1	(27)	Internally calibrated
Seller flow cost	k	33.03	(3.8)	Internally calibrated
Cost convexity	μ	5.658	(0.073)	Internally calibrated
Matching elasticity	γ	0.5454	(0.048)	Internally calibrated

Table 1. Calibrated parameters. Externally calibrated parameters appear in the upper panel.

Table 2 compares the ten targets with their model counterparts. The fit is closest for the ERL distribution among owners. Mean ERL is 12.11 years in the model and 12.43 in the data. The standard deviation is 2.91 against 3.01 and each quartile is within half a year of the data. Mean ERL at purchase is 16.67 years against 16.28 and mean vehicle age is 5.07 years against 4.59. The adjustment rate is too high in the model at 0.110 per year against 0.084. The fraction with zero vehicles is 0.094 against 0.10 and price dispersion is 0.32 against 0.34. Log-price skewness has the wrong sign at 0.71 against -0.51. Buyers close to failure search at high market tightness and pay a large premium for the sellers' expected wait. Those purchases form a right tail of model transaction prices where the data have a left tail.

The untargeted P_{10} and median of ERL are within half a year of their data values. The standard deviation of ERL at purchase is 0.13 years in the model against 1.44 in the data, because buyers' target qualities lie in a narrow band below $\bar{x}$, so the model understates the dispersion of durability across new purchases. For the same reason the transaction-weighted slope of log price on purchased

Moment	Data	Model
Targeted moments		
Mean ERL (years)	12.43	12.11
SD of ERL (years)	3.01	2.91
P25 of ERL (years)	10.25	10.03
P75 of ERL (years)	14.91	14.42
Adjustment rate (annual)	0.08	0.11
Fraction with zero vehicles	0.10	0.09
Price dispersion (SD log p)	0.34	0.32
Log-price skewness	-0.51	0.71
Mean vehicle age (years)	4.59	5.07
Mean ERL at purchase (years)	16.28	16.67
Untargeted moments		
P10 of ERL (years)	8.24	7.93
Median ERL (years)	12.68	12.39
SD of ERL at purchase (years)	1.44	0.13
Price–durability slope χ	0.025	—

Table 2. Model fit and external validation.

ERL is undefined in the model (Appendix D).

6.2 A permanent decline in the discount rate

Between 1970 and 2016 the U.S. natural interest rate r^* fell from 4.16 to 0.90 percent, and over the same period the average age of U.S. passenger cars, which combines vehicle survival and upgrade timing, rose from 5.6 to 11.6 years (Holston et al., 2023, Davis, 2000, Bureau of Transportation Statistics, 2021). New cars also became more durable. The median lifetime of a new automobile rose from 11.3 years for the 1970 model year to 12.2 years for 1980 and 14.0 years for 1990, and the median lifetime of a new light truck moved from 16.8 to 15.7 and 15.2 years (Davis, 2000).[5] The natural interest rate is a real rate, and in a stationary equilibrium of the model the interest rate equals the discount rate ρ, so I read the trend decline in r^* as a decline in ρ. I compare stationary equilibria before and after a permanent decline in the discount rate and compute the transition between them. The calibrated economy of Section 6.1, with annual discount factor 0.96, is the stationary equilibrium after the decline, and the stationary equilibrium before the decline has annual discount factor 0.92 and the same remaining parameters, a decline in ρ of 4.2 percentage

[5] Appendix D describes these series.

	Stationary equilibrium			Change from $\beta = 0.92$ to 0.96		
Outcome	$\beta = 0.92$	Fixed quality	Endogenous quality	Patience	Prices	Quality
Adjustment rate	0.109	0.119	0.110	+0.009	+0.001	-0.008
Owners' replacement rate	0.092	0.104	0.096	+0.011	+0.001	-0.008
Mean vehicle age (years)	5.094	4.834	5.072	-0.234	-0.026	+0.238
Durable expenditure	11.551	12.782	13.479	+1.359	-0.128	+0.697
Average transaction price	106.438	107.842	122.061	+3.228	-1.824	+14.219
Mean ERL (years)	11.544	11.750	12.109	+0.187	+0.019	+0.360
Mean purchased quality (ERL years)	15.903	15.931	16.666	+0.027	+0.002	+0.734
Fraction with zero vehicles	0.112	0.101	0.094	-0.009	-0.001	-0.007
Threshold x_c (ERL years)	13.834	14.191	14.280	+0.446	-0.089	+0.089

Table 3. Stationary effects of a rise in the annual discount factor from $\beta = 0.92$ to $\beta = 0.96$. Fixed quality holds each buyer's target quality at its $\beta = 0.92$ value, and buyers choose market tightness and the timing of search. Patience is the change with buyers at $\beta = 0.96$ and sellers' free-entry prices at $\beta = 0.92$. Prices is the further change when sellers' free-entry prices move to $\beta = 0.96$. Quality is the further change when buyers choose target quality.

points.

Table 3 compares the two stationary equilibria and splits the change into three steps. In the fixed-quality step, each buyer's target quality stays at its value at 0.92 and buyers choose only market tightness and the timing of search at the lower discount rate. The patience column is the part of this step with sellers' free-entry prices still at 0.92, and the prices column is the further change when the free-entry prices move to 0.96. In the endogenous-quality step, the quality column, buyers choose target quality and sellers enter the submarkets they select.

With fixed quality, the lower discount rate moves the threshold x_c from 13.8 to 14.2 ERL years, raises the adjustment rate from 0.109 to 0.119, and lowers mean vehicle age from 5.09 to 4.83 years. Buyers' patience accounts for almost all of this change, and the lower free-entry prices add 0.0008 to the adjustment rate. With endogenous quality, sellers enter at higher qualities and the mean ERL at purchase rises from 15.90 to 16.67 years. Goods of higher quality fail at lower rates and their owners search at lower market tightness, so the adjustment rate falls back to 0.110 and mean vehicle age returns to 5.07 years. The quality response offsets most of the timing response. The adjustment rate ends 1.8 percent above its initial value, against 9.2 percent with fixed quality, and mean vehicle age falls by 0.02 years, against 0.26 years with fixed quality. Mean ERL among owners rises from 11.54 to 12.11 years, and durable expenditure rises from 11.6 to 13.5 because buyers pay for higher quality. Table 4 reports the same pattern across discount factors from 0.92 to 0.96. Mean ERL rises and the fraction with zero vehicles falls, while mean vehicle age and the adjustment rate stay within 0.03 years and 0.002 of their values at 0.92.

	Annual discount factor β		
Outcome	0.92	0.94	0.96
Mean vehicle age (years)	5.09	5.08	5.07
Mean ERL (years)	11.54	11.82	12.11
SD of ERL (years)	2.79	2.85	2.91
Adjustment rate	0.109	0.110	0.110
Fraction with zero vehicles	0.112	0.103	0.094
Price dispersion	0.363	0.342	0.322
Inaction mass	0.225	0.242	0.252

Table 4. Steady-state comparative statics. Each column reports stationary equilibrium outcomes at the annual discount factor $\beta = e^{-\rho}$. The calibration has $\beta = 0.96$.

For a non-owner, the upgrade rate of the purchased good is 0.1273 per year before the decline and 0.1255 after it. At the pre-decline quality, the higher market tightness alone would raise it to 0.1388, so the timing response is +0.0114 and the quality response is −0.0133. Averaged over the stationary purchase flows before the decline, the timing response is +0.0106 and the quality response is −0.0107. In both cases the quality response exceeds the timing response in absolute value. The goods purchased after the decline are upgraded 1.4 percent less often than the goods purchased before it, whereas with fixed quality they would be upgraded 9 percent more often.

Transition dynamics At $t = 0$ the discount factor rises permanently from 0.92 to 0.96. The holdings distribution $g(x, 0)$ and the seller stock start at the old stationary equilibrium, and buyers use the post-shock stationary target policy $\zeta^*_{\text{new}}(x)$ and pay the post-shock posted prices. I assume that the seller stock adjusts toward its post-shock free-entry level with a time constant $\tau_{\text{entry}} = 0.25$ years, net of sales.[6]

Mean ERL rises from 11.54 years in the old stationary equilibrium to 12.11 years in the new one, and the fraction with zero vehicles falls from 0.112 to 0.094 (Figure 3, Table 5). Buyers who were in the inaction region at 0.92 enter the action region at 0.96 and upgrade during the first years, while the seller stock in the new submarkets builds up. The adjustment rate peaks at 0.123 after 0.80 years, 12 percent above its new stationary value of 0.110, and durable expenditure peaks at 15.29 after 0.80 years, 13 percent above its new stationary level of 13.48 and 32 percent above its old level of 11.55.

[6] Appendix **??** states the laws of motion of the holdings distribution and the seller stock, and Appendix B.2 varies τ_{entry}.

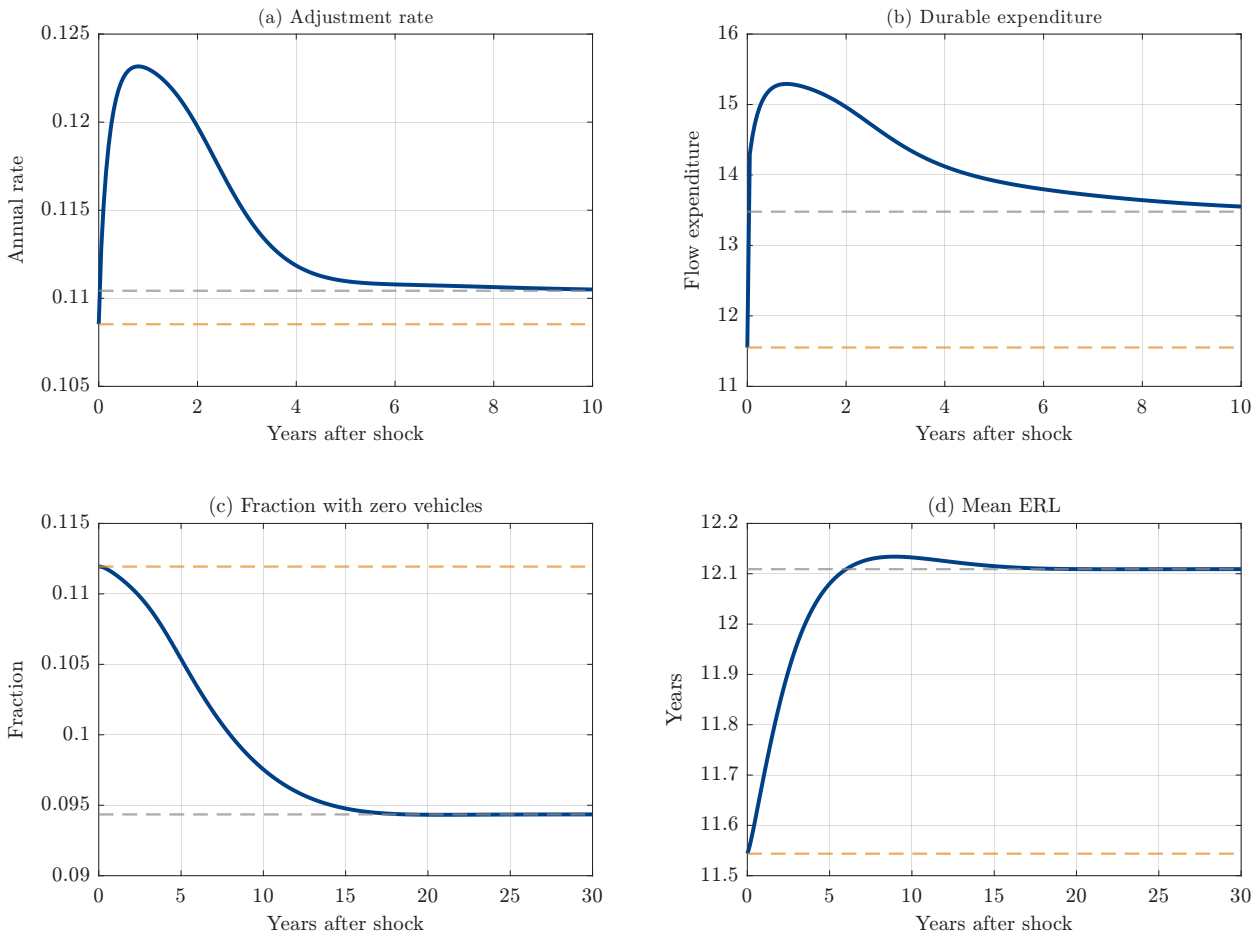


Figure 3. Fixed-policy seller-stock transition after a permanent rise in the discount factor from 0.92 to 0.96. The panels show the annual adjustment rate (a), durable expenditure (b), the fraction with zero vehicles (c), and mean ERL (d). Dashed orange and gray lines mark the old and new stationary equilibria, respectively.

Figure 4 repeats the transition with each buyer's target quality held at its value at 0.92, so that only the timing response operates, and compares it with the transition of Figure 3. The two paths coincide during the first year, and the adjustment rate peaks at 0.123 after 0.80 years in both, because the goods purchased right after the decline are still a small part of holdings. They separate afterwards. With fixed quality the adjustment rate settles at 0.119 and mean vehicle age at 4.83 years. With endogenous quality the goods purchased after the decline survive longer, so the adjustment rate returns to 0.110 within five years and mean vehicle age returns to 5.07 years within twenty years. The timing response is demand today. The quality response is demand tomorrow, and it arrives as the stock of goods purchased after the decline accumulates.

	Old SS	Years after shock					New SS
Outcome	(β=0.92)	t=0.5	t=1	t=2	t=5	t=10	(β=0.96)
Mean ERL (years)	11.54	11.61	11.70	11.85	12.08	12.13	12.11
Adjustment rate	0.109	0.123	0.123	0.120	0.111	0.110	0.110
Durable expenditure	11.551	15.232	15.276	14.963	13.919	13.552	13.479
Fraction with zero vehicles	0.112	0.112	0.111	0.110	0.105	0.098	0.094

Table 5. Fixed-policy seller-stock transition after a permanent decline in the discount rate from $\rho = -\ln(0.92)$ to $\rho = -\ln(0.96)$. Buyers use the post-shock stationary target policy and posted prices. The holdings distribution and seller stock evolve with $\tau_{\text{entry}} = 0.25$ years. Durable expenditure is the purchase flow times transaction prices.

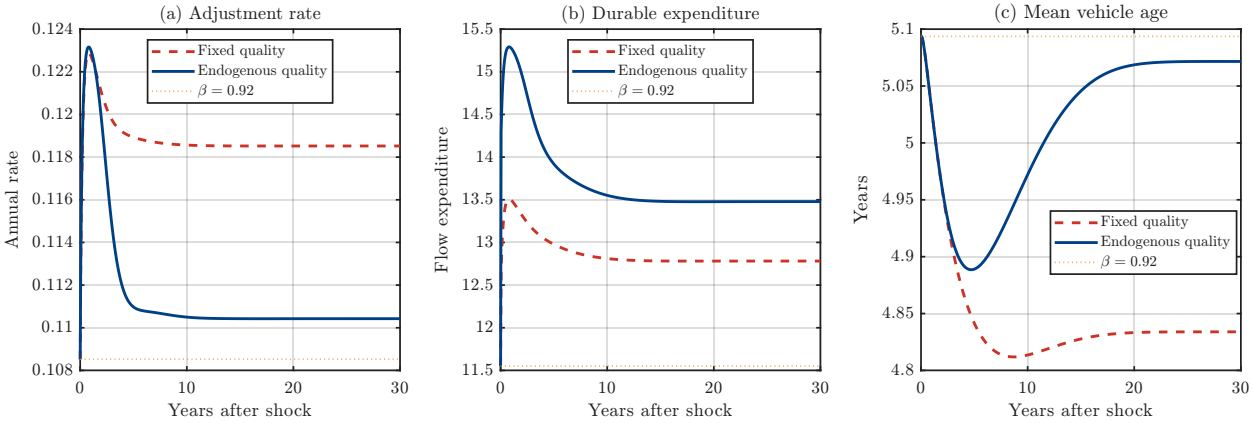


Figure 4. Transition with fixed and with endogenous quality after a permanent rise in the discount factor from 0.92 to 0.96. The dashed line holds each buyer's target quality at its value at 0.92; the solid line is the transition of Figure 3. The panels show the annual adjustment rate (a), durable expenditure (b), and mean vehicle age (c). The dotted line marks the stationary equilibrium at 0.92.

7 Conclusion

I develop a general-equilibrium model of durable goods with directed search and establish three results. The optimal policy partitions holdings into an action region and an inaction region, so adjustment is lumpy with a zero fixed cost, and the stationary distribution of holdings is unique with a mass of non-owners at zero. Buyers in the action region sort across submarkets by the quality of their current good. A buyer close to failure targets lower quality at higher market tightness, a buyer with a better good targets higher quality at lower market tightness, and sellers enter where buyers search, so market tightness decreases in current quality through free entry. A continuum of qualities is traded, and the free-entry price schedule generates dispersion in transaction prices.

I then use the model as a laboratory to study the effect of a lower discount rate. A lower discount rate raises market tightness at a given quality, so buyers upgrade sooner, and sellers supply higher quality and the goods purchased stay in ownership longer. In the calibrated economy, a counter-

factual model with fixed quality predicts 9 percent more upgrades and a stock of goods 0.26 years younger after a 4-point decline in the discount rate. With endogenous quality the goods purchased are upgraded 1.4 percent less often and mean vehicle age falls by 0.02 years. Durable expenditure overshoots along the transition, and the two economies part ways after the first year, as the more durable goods bought after the decline accumulate in the stock. The supply of quality absorbs the demand response that a fixed-quality model attributes to upgrading. Thus, the experiment shows that a model of durable goods that does not account for endogeneous quality provision overestimates the long-run adjustment frequency, while underestimating the expenditure.

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Appendix

A Omitted Proofs and Lemmas

A.1 Proof of Theorem 1

I use $X = [0, \bar{x}]$ and the Banach space $C(X)$ with the sup norm. The free-entry price gives the expected payment flow

$$r(\theta)p(\zeta, \theta) = r(\theta)c(\zeta) + \theta[\rho c(\zeta) + k].$$

This expression is continuous at $\theta = 0$, where it is zero. I first restrict market tightness to $[0, \Theta]$, where $\Theta > \bar{r}d/(\rho k)$.

I construct the holdings process from a Poisson stream of purchase opportunities at rate $\bar{r}$, each of which the buyer accepts with probability $r(\theta)/\bar{r}$ and converts into a good of quality ζ, and from failures that arrive at the integrated rate along the depreciation path and send the buyer to zero. Although the failure rate is unbounded near zero, the process is well defined, because a buyer at zero faces zero failure risk and every failure after the first has to be preceded by a purchase, so over a finite period the number of failures is at most the number of purchase opportunities plus one.

I denote quality at date t by X_t and expectation from initial quality x by $\mathbb{E}_x$. For $h > 0$ and $W \in C(X)$, the finite-horizon Bellman operator is

$$(T_h W)(x) = \sup_{\{\zeta_t, \theta_t\}} \mathbb{E}_x \left[\int_0^h e^{-\rho t} \left\{ d\mathbf{1}_{\{X_t > 0\}} - r(\theta_t)c(\zeta_t) - \theta_t[\rho c(\zeta_t) + k] \right\} dt + e^{-\rho h} W(X_h) \right]. \quad (16)$$

For $W, Z \in C(X)$, the same policy and event realizations give

$$\|T_h W - T_h Z\|_\infty \leq e^{-\rho h} \|W - Z\|_\infty. \quad (17)$$

Continuity of $T_h W$ at positive qualities follows from a coupling. Two buyers start at positive qualities x and x' and use the same actions and the same purchase opportunities. Their states coincide from the first purchase or failure onward. Independent randomization lets one buyer reproduce the history of the other whenever the two histories differ. Before the common event both depreciation paths lie in a positive compact interval over $[0, h]$ on which λ is uniformly continuous.

The probability that exactly one buyer fails before the common event therefore tends to zero as $x' \to x$. Uniform continuity of W bounds the gap between the terminal values. Both bounds hold uniformly across policies.

Continuity at zero follows from a second coupling. An owner at $x > 0$ and a buyer at zero use the same actions and the same purchase opportunities. Their payments coincide and their states merge at the owner's failure or at a common purchase. Before that event the owner collects the extra flow d. The failure rate along the owner's path is at least $\lambda(x)$ and the expected time until the common event is at most $1/\lambda(x)$. Comparison in both directions gives

$$|T_h W(x) - T_h W(0)| \leq \frac{d}{\lambda(x)} + \max_{0 \leq z \leq x} |W(z) - W(0)|.$$

The bound tends to zero as $x \downarrow 0$.

Each T_h is thus a contraction on $C(X)$ with a unique fixed point, and since dynamic programming yields $T_s T_t = T_{s+t}$, that fixed point is common to every T_t and iteration from zero identifies it with the infinite-horizon buyer value V. Payments are nonnegative and the ownership flow is at most d, while zero search earns a nonnegative payoff, so $0 \leq V \leq d/\rho$. Run over the infinite horizon, the coupling with a buyer at zero also gives

$$0 \leq V(x) - U \leq \frac{d}{\rho + \lambda(x)}. \tag{18}$$

Hence $\lambda(x)[V(x) - U] \leq d$. Compactness of the restricted action set makes the search gain D continuous. Stop the dynamic program at the first purchase, failure, or date h. Before this event, quality is $xe^{-\delta t}$. For any sequence of search actions, write

$$H_t = \int_0^t [\rho + \lambda(xe^{-\delta s}) + r(\theta_s)]\, ds.$$

The first-event identity is

$$V(x) = \sup_{\{\zeta_t, \theta_t\}} \Bigg\{ \int_0^h e^{-H_t} \big[d - r(\theta_t) c(\zeta_t) - \theta_t [\rho c(\zeta_t) + k] + \lambda(xe^{-\delta t}) U + r(\theta_t) V(\zeta_t) \big] dt + e^{-H_h} V(xe^{-\delta h}) \Bigg\}. \tag{19}$$

On a positive compact interval, the rates and payoffs in this identity are uniformly bounded. Subtraction of $V(xe^{-\delta h})$ bounds $|V(x) - V(xe^{-\delta h})|$ by a constant times h, uniformly for sufficiently small h. Thus the value along a depreciation path is locally Lipschitz and absolutely continuous.

At a point where V is differentiable along the path, the first-event identity reduces to the HJB, with one inequality from holding fixed an action that attains the current maximum and the other from the fact that the maximum bounds every action, and continuity of V together with the local bounds keeps the errors uniform across actions. Absolute continuity then gives

$$V(xe^{-\delta t}) - V(x) = \int_0^t \left[\rho V(xe^{-\delta s}) - d + \lambda(xe^{-\delta s})[V(xe^{-\delta s}) - U] - D(xe^{-\delta s})\right] ds. \tag{20}$$

The integrand is continuous at every finite date. Therefore V is continuously differentiable at positive qualities and has a left derivative at $\bar{x}$. Equation (20) yields (1). A buyer at zero stays there until a purchase, so $\rho U = D(0)$. A measurable stationary maximizer exists because the objective is continuous on a compact action set. Discounted integration of the HJB along the path of this policy shows that it attains V. The integration first stops the process when it leaves a positive compact interval or reaches a finite horizon. Purchase opportunities arrive at the bounded rate $\bar{r}$ and each failure after the first requires a purchase before it. The expected number of events is therefore finite. Bound (18) controls the failure term. The interval then grows and the horizon recedes. Bounded value and bounded expected payments justify these limits and the discounted terminal value tends to zero.

For every $\theta > \Theta$, the unrestricted search objective satisfies

$$r(\theta)[V(\zeta) - c(\zeta) - V(x)] - \theta[\rho c(\zeta) + k] \leq \frac{\bar{r}d}{\rho} - k\theta < 0,$$

while the choice $\theta = 0$ gives zero, so the same HJB verifies V against every admissible policy with unrestricted market tightness. A policy with finite expected discounted payments admits the verification limits. A policy with infinite expected discounted payments has value $-\infty$. The restriction of market tightness to $[0, \Theta]$ therefore leaves the buyer value as it is. The same verification applied to any bounded continuous solution of the HJB with the stated boundary value at zero shows that this solution equals V. Next, fix any measurable stationary search policy. For $h > 0$, every

initial state has probability at least

$$\varepsilon_h = e^{-\bar{r}h}\left[1 - \exp\left\{-\int_0^h \lambda(\bar{x}e^{-\delta t})\,dt\right\}\right] > 0$$

of being at zero at date h. For an initial owner this is the event of zero purchase opportunities together with a failure before h, and a buyer who starts at zero stays there on the same event. For two initial probability distributions μ and ν and the transition probabilities P_h over h, the total variation distance therefore satisfies

$$\|\mu P_h - \nu P_h\|_{\mathrm{TV}} \leq (1 - \varepsilon_h)\|\mu - \nu\|_{\mathrm{TV}}.$$

The space of probability distributions on the state space is complete under total variation and P_h has a unique invariant distribution. Iteration of the bound gives convergence from every initial distribution at a geometric rate. The invariant distribution places mass at least ε_h at zero.

With the price supplied by free entry in (4) and the seller mass in each selected submarket equal to its buyer mass times market tightness, a stationary optimal policy and its invariant distribution form an equilibrium. When the active choices are unique, so are the active prices and the stationary allocation, with targets at zero search recorded as zero.

Owners with higher-quality goods face lower failure rates while following higher depreciation paths. They have access to the same search offers. A buyer with a better good can follow the same search choices as a buyer with a worse good and receive durable services through additional dates.

Lemma A.1 (Monotonicity of V)**.** *The value function V is strictly increasing on $[0, \bar{x}]$.*

Proof. Take $0 \leq x_1 < x_2 \leq \bar{x}$. Couple two buyers with the same search choices, payments, and purchase opportunities. The buyer at x_2 uses independent randomization to reproduce the history of a policy starting from x_1. Purchases occur together and reset both qualities to the same target. Before that event, proportional depreciation preserves their order. Common failures occur at the better good's failure rate. Additional failures affect the worse good at the difference between their failure rates. The better good supplies weakly more ownership flow along every coupled path. Taking suprema gives $V(x_2) \geq V(x_1)$.

For $x_1 > 0$, fix $h > 0$. Continuity and strict decrease of λ supply finite constants L and $\eta > 0$

such that

$$\lambda(x_2 e^{-\delta t}) \leq L, \qquad \lambda(x_1 e^{-\delta t}) - \lambda(x_2 e^{-\delta t}) \geq \eta \qquad (0 \leq t \leq h).$$

Zero purchase opportunities through h, survival of the better good through h, and failure of the worse good by $h/2$ have joint probability at least $e^{-(\bar{r}+L)h}(1 - e^{-\eta h/2}) > 0$. The better good supplies the extra flow d throughout $[h/2, h]$ on this event. The expected discounted gain has a strictly positive lower bound uniform across the lower buyer's policies. Thus $V(x_2) > V(x_1)$.

For $x_1 = 0$, bound the better good's failure rate through h by L. The extra expected flow before a purchase or failure is at least $d \int_0^h e^{-(\rho+\bar{r}+L)t} dt > 0$. Hence $V(x_2) > V(0)$. ■

Lemma A.1 gives $D(\bar{x}) = 0$. Drift remains negative at $\bar{x}$, so its one-sided HJB is

$$\rho V(\bar{x}) = d - \delta \bar{x} V'_-(\bar{x}).$$

The upper-boundary value is strictly below d/ρ. A failure before h and zero purchase opportunities before $2h$ have probability at least

$$e^{-2\bar{r}h}\left[1 - \exp\left\{-\int_0^h \lambda(\bar{x}e^{-\delta t})\, dt\right\}\right] > 0.$$

Ownership flow is zero throughout $[h, 2h]$ on this event. Payments are nonnegative, so $V(\bar{x}) < d/\rho$. Its derivative is determined by the one-sided HJB.

An owner who keeps the current good until failure receives d while it survives and continuation value U at failure. The failure time τ_x and its survival probability are defined in Section 2. The auxiliary value is

$$\hat{V}(x) = \mathbb{E}\left[\int_0^{\tau_x} e^{-\rho t} d\, dt + e^{-\rho \tau_x} U\right] = U + (d - \rho U) \int_0^\infty \exp\left\{-\rho t - \int_0^t \lambda(x e^{-\delta s})\, ds\right\} dt. \tag{21}$$

The integral is finite and tends to zero as $x \downarrow 0$. It yields $\hat{V}(0) = U$ and satisfies the ownership-until-failure HJB at positive qualities.

A.2 Proofs of Lemmas 5 and 6 and Proposition 2

Write $V_i(x) = V(x;\rho_i)$, $U_i = V_i(0)$, and let (ζ_1, θ_1) be a stationary optimal policy at ρ_1. Its net flow payoff at discount rate ρ_i is

$$d\mathbf{1}_{\{x>0\}} - r(\theta_1)c(\zeta_1) - \theta_1[\rho_i c(\zeta_1) + k].$$

I write $\mathbb{E}^1_x$ for expectation under this policy from initial quality x. The HJB for V_1 and discounted integration along this policy at ρ_2 give

$$V_2(x) \geq V_1(x) + (\rho_1 - \rho_2)\,\mathbb{E}^1_x \int_0^\infty e^{-\rho_2 t}[V_1(X_t) + \theta_1(X_t)c(\zeta_1(X_t))]dt.$$

The bounded-value verification in the preceding proof justifies the integration. The expected discounted flow utility until failure decreases in the discount rate, so the restriction on production cost holds at both discount rates. Lemma 2 at ρ_1 gives $U_1 > 0$, and Lemma A.1 gives $V_1 \geq U_1$. The integral is therefore strictly positive, which gives $V_2 > V_1$. For the threshold, take a maximizer $\widehat{\zeta}$ of $V_1(\zeta) - c(\zeta)$. Lemma 2 gives

$$V_1(\widehat{\zeta}) - c(\widehat{\zeta}) = V_1(x_c(\rho_1)), \qquad \widehat{\zeta} > x_c(\rho_1).$$

Strict increase of $V_2 - V_1$ implies

$$V_2(\widehat{\zeta}) - c(\widehat{\zeta}) - V_2(x_c(\rho_1)) > 0.$$

A buyer at the old threshold therefore searches at ρ_2. Lemma 2 gives $x_c(\rho_2) > x_c(\rho_1)$.

For Lemma 6, write $a(\theta;\rho) = 1 + \rho/q(\theta)$ for the factor in (8). Strict concavity of r with $r(0) = 0$ makes q strictly decreasing, so a is strictly increasing in θ and in ρ. For parts (i) and (iii), fix a continuous strictly increasing W, let $a > a' > 0$, and let ζ and ζ' be the maximizers of $W(\cdot) - a\,c(\cdot)$ and $W(\cdot) - a'c(\cdot)$. Optimality of each target against the other gives $W(\zeta') - a'c(\zeta') \geq W(\zeta) - a'c(\zeta)$ and $W(\zeta) - a\,c(\zeta) \geq W(\zeta') - a\,c(\zeta')$. Adding the two inequalities yields $(a - a')[c(\zeta') - c(\zeta)] \geq 0$, so $\zeta' \geq \zeta$. Equal interior targets would satisfy the first-order conditions $W'(\zeta) = a'c'(\zeta)$ and $W'(\zeta) = a\,c'(\zeta)$ with $c'(\zeta) > 0$, which contradicts $a > a'$. Hence $\zeta' > \zeta$. Part (i) takes $a = a(\theta;\rho_1)$

and $a' = a(\theta; \rho_2)$ at a fixed θ, and part (iii) takes $a = a(\theta; \rho)$ and $a' = a(\theta'; \rho)$ with $\theta > \theta'$.

For part (ii), fix $\theta > 0$, write $a_i = a(\theta; \rho_i)$, so $a_1 > a_2$, and let ζ_i be the maximizer of $V_i(\cdot) - a_i c(\cdot)$. Optimality gives $V_2(\zeta_2) - a_2 c(\zeta_2) \geq V_2(\zeta_1) - a_2 c(\zeta_1)$ and $V_1(\zeta_1) - a_1 c(\zeta_1) \geq V_1(\zeta_2) - a_1 c(\zeta_2)$. Suppose $\zeta_2 < \zeta_1$. The second inequality and strict increase of $V_2 - V_1$ yield

$$V_2(\zeta_1) - V_2(\zeta_2) \geq V_1(\zeta_1) - V_1(\zeta_2) \geq a_1[c(\zeta_1) - c(\zeta_2)] > a_2[c(\zeta_1) - c(\zeta_2)],$$

which contradicts the first inequality. Hence $\zeta_2 \geq \zeta_1$. Strict increase of $V_2 - V_1$ gives $V_2' \geq V_1'$ at positive qualities, where both derivatives exist by Theorem 1. Equal interior targets would satisfy $V_2'(\zeta) = a_2 c'(\zeta) < a_1 c'(\zeta) = V_1'(\zeta)$, a contradiction. Hence $\zeta_2 > \zeta_1$.

Write $\mathcal{S}(a; \zeta, \theta)$ for the integrand in (14), the probability that a good purchased at quality ζ is still in ownership at age a. Fix $\zeta, \zeta' \in (0, \bar{x}]$ and tightness policies θ, θ', and write $h(a) = \lambda(\zeta e^{-\delta a}) + r(\theta(\zeta e^{-\delta a}))$ and $h'(a) = \lambda(\zeta' e^{-\delta a}) + r(\theta'(\zeta' e^{-\delta a}))$ for the two exit rates. Write

$$\ln \frac{\mathcal{S}(a; \zeta', \theta')}{\mathcal{S}(a; \zeta, \theta)} = -\int_0^a [h'(s) - h(s)]\, ds.$$

For $a \geq 1$, strict decrease of λ gives $\lambda(\zeta e^{-\delta a}) \geq \lambda(\zeta e^{-\delta}) > 0$, so $\mathcal{S}(a; \zeta, \theta) \leq \exp\{-\lambda(\zeta e^{-\delta})(a - 1)\}$ and the integral in (14) is finite. If $h' \geq h$ at every age, then $\mathcal{S}(\cdot; \zeta', \theta') \leq \mathcal{S}(\cdot; \zeta, \theta)$ and $n(\zeta', \theta') \geq n(\zeta, \theta)$. If $h' < h$ at every age, the survival inequality is strict at every positive age and $n(\zeta', \theta') < n(\zeta, \theta)$.

The timing response takes $\zeta' = \zeta = \zeta_1(x)$, $\theta = \theta_1$ and $\theta' = \theta_2$. The hypothesis $\theta_2 \geq \theta_1$ and strict increase of r imply $h' \geq h$, so $n(\zeta_1(x), \theta_2) \geq n(\zeta_1(x), \theta_1)$.

The quality response takes $\theta = \theta' = \theta_2$, $\zeta = \zeta_1(x)$ and $\zeta' = \zeta_2(x) > \zeta$. At every age, $\zeta' e^{-\delta a} > \zeta e^{-\delta a}$. Strict decrease of λ implies $\lambda(\zeta' e^{-\delta a}) < \lambda(\zeta e^{-\delta a})$, Lemma 1 implies $\theta_2(\zeta' e^{-\delta a}) \leq \theta_2(\zeta e^{-\delta a})$, and r is increasing. Hence $h' < h$ at every age and $n(\zeta_2(x), \theta_2) < n(\zeta_1(x), \theta_2)$. Equation (15) adds and subtracts $n(\zeta_1(x), \theta_2)$.

B Additional Extensions

B.1 State-Independent Failure Rate

Owners receive the same flow utility from every functioning good. Under a constant failure rate each positive quality delivers the same expected services. Depreciation keeps quality positive at every finite date. Owners therefore keep their goods until failure. Buyers at zero prefer lower production cost for the same ownership services.

Lemma B.1. *Suppose $\lambda(x) = \lambda_0 > 0$ for $x > 0$, with zero failure risk at the non-owner state. The buyer's value, with the target supremum over positive qualities, is*

$$V(x) = \frac{d + \lambda_0 U}{\rho + \lambda_0} > U \qquad (x > 0). \tag{22}$$

Owners choose zero search. The value U is the unique solution in $(0, d/\rho)$ of

$$\rho U = \max_{\theta \geq 0} \left\{ r(\theta) \frac{d - \rho U}{\rho + \lambda_0} - k\theta \right\}.$$

Buyers at zero approach their supremum as $\zeta \downarrow 0$. The original positive offered-quality set has an unattained infimum at zero, so 0^+ describes a limiting target. The positive-state value has a jump at zero.

Proof. Every positive quality gives the same ownership flow and failure rate, and buyers have the same search offers. All positive qualities therefore have the same buyer value. An upgrade leaves that continuation value unchanged and requires a positive payment, so owners choose zero search. Their HJB gives

$$(\rho + \lambda_0)V = d + \lambda_0 U, \qquad V - U = \frac{d - \rho U}{\rho + \lambda_0}.$$

For a buyer at zero, the objective at any positive target is

$$r(\theta)[V - U - c(\zeta)] - \theta[\rho c(\zeta) + k].$$

At positive market tightness this expression strictly increases as the positive target falls, since c is

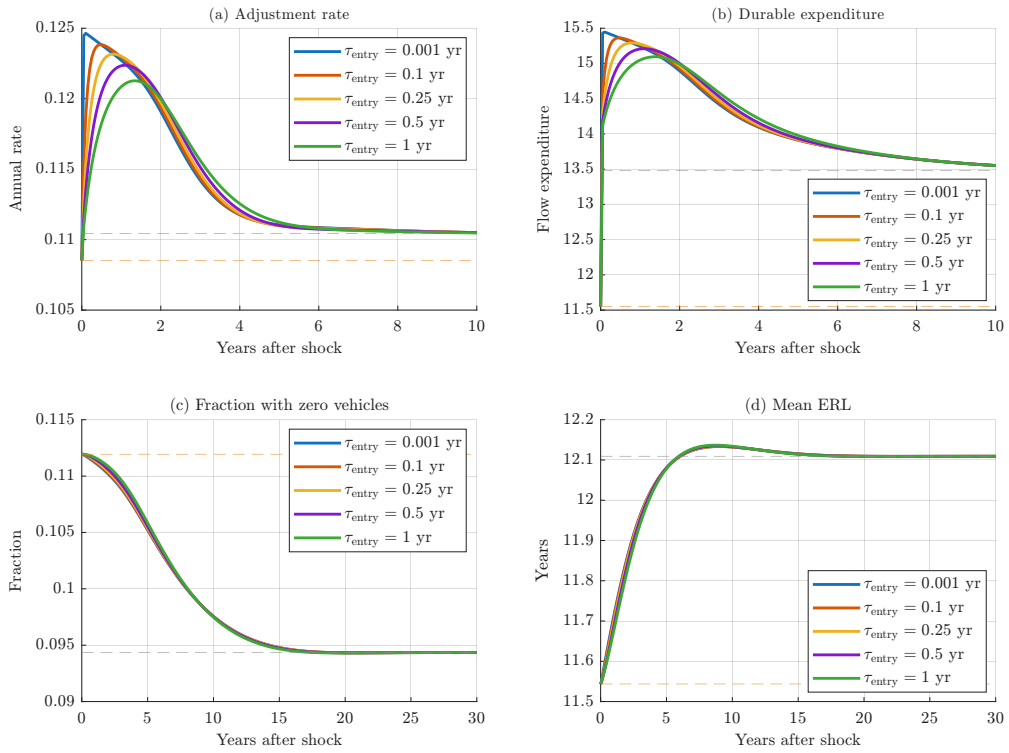


Figure 5. Sensitivity of the fixed-policy seller-stock transition to τ_{entry}. Panels (a)–(d) show the adjustment rate, durable expenditure, fraction with zero vehicles, and mean ERL, respectively. The dashed lines mark the old stationary equilibrium at $\beta = 0.92$ in orange and the new stationary equilibrium at $\beta = 0.96$ in gray.

strictly increasing and $c(\zeta) \to 0$. Taking the supremum over targets yields the scalar equation in the statement. Its right-hand side is continuous and weakly decreasing in U. It is strictly positive at $U = 0$ because $r'(0+) = \infty$, and it is zero at $U = d/\rho$. The left-hand side is strictly increasing. Their unique intersection lies in $(0, d/\rho)$, with a unique positive optimal market tightness by strict concavity of r. Each positive target is strictly dominated by a smaller positive target at that market tightness. Thus the target supremum is approached as $\zeta \downarrow 0$. A purchase at $\zeta = 0$ leaves the buyer at the non-owner state. The limiting target therefore remains distinct from an attained purchase at zero. ■

B.2 Seller-Stock Transition. Sensitivity to τ_{entry}

I vary $\tau_{\text{entry}} \in \{0.001, 0.1, 0.25, 0.5, 1.0\}$ years in Figure 5. The adjustment rate peaks between 0.121 and 0.125 after 0.10 to 1.35 years, and durable expenditure peaks between 15.09 and 15.45 at the same dates. A faster seller-stock adjustment moves both peaks earlier and raises them slightly. At the baseline $\tau_{\text{entry}} = 0.25$ both peaks occur after 0.80 years.

C Horizontal Differentiation

Buyers value goods with characteristics $(\zeta, \kappa) \in [0, \bar{x}] \times [0, \bar{q}]$. Durability ζ is expected residual lifetime, and κ is a horizontal taste characteristic. The good depreciates at rate δ and retains its horizontal taste characteristic throughout its life.

C.1 Buyer's Problem

Owners receive baseline flow utility $d > 0$. The horizontal taste characteristic of the current good is q. Buyers value this characteristic through the scale $d_q > 0$ and curvature $\eta > 0$. A buyer holding a good of quality (x, q) receives flow utility

$$u(x,q) \;=\; d\Big(1 \;+\; d_q\,\frac{q^{1-\eta}}{1-\eta}\Big), \tag{23}$$

I assume the value function is supermodular, with $V_{xq} > 0$.

The buyer's HJB equation is

$$\rho\, V(x,q) \;=\; u(x,q) \;-\; \delta\, x\,\frac{\partial V}{\partial x} \;-\; \lambda(x)\big[V(x,q) - U\big] \;+\; D(x,q), \tag{24}$$

where

$$D(x,q) \;=\; \max\Big\{\max_{\zeta,\kappa,\theta}\; r(\theta)\big[V(\zeta,\kappa) - V(x,q) - p(\zeta,\kappa)\big],\; 0\Big\}. \tag{25}$$

C.2 Seller's Problem and Free Entry

Sellers post prices p in submarkets indexed by (ζ, κ). The coefficient c_q sets the production cost scale for the horizontal taste characteristic, and μ_q is its cost exponent. The coefficient $c_\times > 0$ gives complementary production costs across the two characteristics. The production cost is

$$C(\zeta,\kappa) \;=\; c\,\zeta^{\mu} \;+\; c_q\,\kappa^{\mu_q} \;+\; c_\times\,\zeta^{\mu}\,\kappa^{\mu_q}, \tag{26}$$

with $C_{\zeta\kappa} > 0$. An operating seller pays the flow cost $\bar{\kappa}$. Sellers post the free-entry price

$$p(\zeta, \kappa) \;=\; \frac{(\rho + q(\theta))\, C(\zeta, \kappa) + \bar{\kappa}}{q(\theta)}. \tag{27}$$

C.3 Optimality Conditions

Buyers choose durability and the horizontal taste characteristic in the same submarket. An increase in either characteristic raises the value after purchase and changes production cost. The seller pays that cost before the match. The seller faces the same expected wait to recover the production cost of either characteristic.

A buyer choosing an interior target equates the marginal value of each characteristic with its marginal price. The seller's expected wait enters both marginal prices through the same factor. That factor cancels from their ratio, so the buyer's relative marginal values equal the relative marginal production costs. Complementary production costs make the marginal cost of either characteristic rise with the other.

The first-order conditions yield

$$\left.\frac{\partial V/\partial \zeta}{\partial V/\partial \kappa}\right|_{(\zeta^*, \kappa^*)} \;=\; \frac{C_\zeta(\zeta^*, \kappa^*)}{C_\kappa(\zeta^*, \kappa^*)}, \tag{28}$$

D Data Construction and Empirical Moments

D.1 Natural Interest Rate and Vehicle Age

I average the Federal Reserve Bank of New York's quarterly U.S. natural interest rate estimates r^* based on Holston et al. (2023) by calendar year over 1970–2016, using the workbook downloaded on October 31, 2026, which ends in 2026Q2. Passenger-car age comes from the mean automobile-age column of Table 6.8 in Davis (2000) for 1970–1994, from the passenger-car column of the archived BTS Table 1-26 dated July 17, 2015 for 1995–2014 (Bureau of Transportation Statistics, 2015), and from Table 1-26 in Bureau of Transportation Statistics (2021) for 2015–2016. These sources report the mean age of automobiles in use and the average age of passenger cars in operation, respectively, and both report 8.4 or 8.5 years for 1995. The median lifetimes of new automobiles and light trucks by model year come from Tables 6.9 and 6.10 of Davis (2000). The annual CSV in the replication package records the source URL and table location for each observation.

D.2 Panel Study of Income Dynamics

The PSID (Panel Study of Income Dynamics, 2021) follows a nationally representative sample of households annually through 1997 and biennially thereafter. Households report up to three vehicles in the vehicle module. I use the 1999–2011 waves, which record whether each vehicle was new or used at acquisition, and process the raw files with the PSID-SHELF harmonization pipeline of Pfeffer et al. (2024). Buyers in the model acquire new goods, so the vehicle moments use vehicles acquired new.

I map vehicle age and make to ERL with the survival schedules of Davis (2000) and the durability of each make estimated in the PSID, then compute moments on the vehicles acquired new.

D.3 Stage 1. Survival, Durability by Make, and ERL

Tables 6.9 and 6.10 of Davis (2000) report survival and scrappage rates by vehicle age for automobiles and light trucks of model year 1990, with zero scrappage below age four. The light-vehicle survival function $S_0(a)$ is the average of the two schedules with equal weights. A vehicle of make m has survival $S_m(a) = S_0(a)^{\theta_m}$. I estimate θ_m from the PSID waves 1999–2019 by comparing the number

of vehicles of make m and model year v per household in two consecutive waves, which measures the share of the cohort that survives two years. Makes with fewer than 1,000 vehicle-years are pooled, and θ_m is normalized to a vehicle-weighted mean of one. A vehicle of make m that survives to age a has ERL

$$X_m(a) \;=\; \mathbb{E}[T - a \mid T > a] \;=\; \frac{1}{S_m(a)} \int_a^{\infty} S_m(u)\, du. \tag{29}$$

The ERL of a new vehicle ranges across makes from 12.3 to 17.9 years. The top quality in the model, $\bar{x}$, is the ERL of a new vehicle of the most durable make, and $\sigma = X_{\max}(0)/\bar{x}$ converts state units into years.

The failure rate in the model is a function of residual lifetime. At each ERL level, I average the hazard $\theta_m\, h_0(a)$ over the makes and ages that have that ERL, weighted by the number of vehicles of the make and the surviving share $S_m(a)$, and I impose that the average is weakly decreasing in ERL by pooling adjacent violators. The schedule runs from an ERL of 1.95 years, where the failure rate equals its cap $\bar{\lambda} = 0.50$ per year, to 17.9 years, where it equals 0.0004 per year.

D.4 Stage 2. Moments on the Representative Panel

I map each vehicle–year observation from make and age to ERL via $X_{m(i)}(a_{i,t})$. The targeted ERL moments among owners are the mean and standard deviation, together with P_{25} and P_{75}. ERL at purchase is $X_{m(i)}(0)$ for vehicles acquired new and observed within a year of their model year, and its mean across purchases is targeted. Its standard deviation across purchases is an untargeted moment. The share of households that acquire a new vehicle between two consecutive biennial waves is $\bar{A}$. I annualize the adjustment rate as $\bar{A}/2$ and set the fraction with zero vehicles to 0.10 following Gavazza and Lanteri (2021). Price dispersion, log-price skewness and mean vehicle age complete the ten targeted moments.

D.5 Hedonic price regression

I estimate the relation between log prices and ERL on the vehicles acquired new via

$$\log p_{i,t} = \alpha_0 + \chi X_{m(i)}(a_{i,t}) + \phi_{m(i)} + \lambda_t + \varepsilon_{i,t}, \tag{30}$$

where $p_{i,t}$ is the reported transaction price. I include manufacturer fixed effects $\phi_{m(i)}$ and year fixed effects λ_t. I measure price dispersion by the standard deviation of $\varepsilon_{i,t}$, which is 0.342. The skewness of $\log p_{i,t}$ is -0.514.

The price–durability slope compares the prices of new vehicles that differ in durability at purchase. I regress the log purchase price of vehicles acquired new and observed within a year of their model year on $X_{m(i)}(0)$, with year fixed effects. The slope is $\hat{\chi}_{\text{new}} = 0.025$ (standard error 0.003). Its model counterpart is the transaction-weighted regression of log price on the ERL of the purchased good, which is defined when the standard deviation of purchased ERL exceeds a quarter of a year. At the calibrated parameters that standard deviation is 0.13 years, so Table 2 leaves the model slope blank. The posted price schedule at a given market tightness rises with quality, with a slope of 0.20 at the tightness of the average purchase.

E Functional Forms and Identification

Matching technology. The quantitative model uses the bounded buyer meeting rate

$$r(\theta) \;=\; \frac{A\,\theta^{\gamma}}{1 + B\,\theta^{\gamma}}, \tag{31}$$

where $A > 0$ is matching efficiency and $B > 0$ is matching congestion. The matching elasticity is $\gamma \in (0, 1)$. A seller meets a buyer at rate

$$q(\theta) = \frac{r(\theta)}{\theta} = \frac{A\,\theta^{\gamma-1}}{1 + B\,\theta^{\gamma}}. \tag{32}$$

Buyers meet sellers at a rate with decreasing returns in market tightness. The aggregate meeting function has constant returns and satisfies $br = sq$. I use this seller meeting rate in the buyer problem and free-entry condition, as well as in seller-stock accounting.

Failure rate. The failure rate $\lambda(x)$ is the schedule of Appendix D, evaluated at ERL $= \sigma x$. It equals $\bar{\lambda} = 0.50$ per year for $x \leq x_{\lambda}$, where $\sigma x_{\lambda} = 1.95$ years, and decreases weakly in x above x_{λ} to 0.0004 per year at $\bar{x}$. The annual discount factor is $\beta = e^{-\rho}$.

Production cost and flow utility. Sellers pay production cost $c(\zeta) = c\,\zeta^{\mu}$ with $\mu > 1$, and I normalize its scale to $c = 0.01$. Owners receive flow utility $d > 0$. Their goods depreciate according to $\dot{x} = -\delta x$. I map the grid upper bound $\bar{x}$ to the ERL of a new vehicle of the most durable make through ERL $= \sigma x$ (Appendix D).