

# Coordination in Technology Adoption: Networks, Innovation, and Growth

Nicolò Ceneri\*  
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## Abstract

I develop a model in which adoption and innovation mutually reinforce each other, making an economy's rate of innovation depend on whether sectors coordinate on adoption. When complementary sectors coordinate on adopting the frontier technology, TFP increases, innovators earn more and invest more in research; the further ahead research advances the frontier, the greater the gains from coordinating on its adoption. Conversely, when sectors persistently fail to coordinate, research falters, the frontier stalls, and so do the incentives to coordinate: an expected failure to adopt becomes a permanent loss of innovation. I measure adoption complementarities between U.S. sectors from the technology content of job postings and identify their strength using idiosyncratic hiring shocks at large firms. The estimated complementarities, together with the losses from adopting alone, are strong enough for coordination to fail. In the calibrated economy, coordinating on adoption every period raises annual TFP growth by 0.39 percentage points. Adoption subsidies target sectors whose adoption supports many other sectors and whose losses from adopting alone are small, such as semiconductors, software, and data centers.

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\*Department of Economics, New York University, 19 W 4th St., 10012, New York, NY. Email: [nicolo.ceneri@nyu.edu](mailto:nicolo.ceneri@nyu.edu). I am deeply grateful to Guido Menzio, Jess Benhabib, Paolo Martellini, and Venky Venkateswaran for their invaluable guidance. I am grateful to Gadi Barlevy, Corina Boar, Jaroslav Borovička, Alessandro Dovis, Joel Flynn, Simon Gilchrist, Jeremy Greenwood, Tarek Hassan, Christian Hellwig, David Hémous, Ricardo Lagos, Danial Lashkari, Francesco Lippi, Ernest Liu, Guido Lorenzoni, Virgiliu Midrigan, Ezra Oberfield, Aleksei Oskolkov, Diego Perez, Alessandra Peter, Michael Peters, Edouard Schaal, Lorenzo Vacca, Iván Werning, and the participants of the NYU Stern Macroeconomics Seminar for helpful comments. All errors are mine.

# 1 Introduction

Adopting a technology requires coordination across sectors. A technology that is not profitable for one sector to adopt in isolation may become profitable once enough complementary sectors adopt it as well. For the same technologies, two extremely different outcomes can arise: one in which adoption stalls and one in which it takes off. This is the logic of the *big-push* literature (Rosenstein-Rodan, 1943; Murphy et al., 1989; Buera et al., 2021). In that tradition, and in models of technology adoption and diffusion (Comin and Hobijn, 2010; Comin and Mestieri, 2018; Alvarez et al., 2023), the technology available for adoption is taken as given. Coordination determines whether existing technology is adopted, but not how fast the technological frontier itself advances. Yet the frontier advances because innovators expect to earn from the adoption of its technology (Bresnahan and Trajtenberg, 1995; Anzoategui et al., 2019). This paper studies the feedback between adoption and innovation. Ex-post coordination determines the market for innovation: whether sectors coordinate on using the frontier determines the return to advancing it. Innovation, in turn, changes the incentives to coordinate on adoption. As the frontier moves further ahead of the technology sectors already use, adopting it pays more, and each sector's adoption does more to raise adoption in complementary sectors. An economy can innovate less because it fails to adopt the technology it already has.

The shift from dial-up Internet to broadband illustrates the mechanism. In the spring of 2001, the *Wall Street Journal* described broadband as caught in a “chicken-and-egg” problem. On the one hand, high-bandwidth applications were worth little on slow connections, so content providers waited until broadband reached a larger audience; on the other hand, fast connections were worth little without such applications, making broadband difficult to sell until there was more to use it for.<sup>1</sup> The technologies already existed; however, each side was waiting for the other to use them. The coordination failure, in turn, affected innovation. Intel, whose faster processors had less value on slow connections, described affordable broadband as a fundamental driver of the next generation of microprocessors and lobbied for its deployment.<sup>2</sup> The same coordination problem that governed the use of existing technologies thus affected the return to developing new ones. More generally, this paper addresses two questions. Under what conditions does a technology that already exists go unused? How large are the aggregate consequences on innovation and growth?

I answer these questions in three steps. First, I develop a dynamic model in which adoption is a coordination game across sectors and innovators choose how rapidly to advance the frontier. Second, I measure the adoption network and the strength of these complementarities using the technology content of U.S. job postings. Third, I embed these estimates in the dynamic model and quantify the consequences of coordination for innovation and long-run growth.

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<sup>1</sup>Jared Sandberg, “Broadband’s Chicken-and-Egg Bind,” *Wall Street Journal*, April 2, 2001, B1.

<sup>2</sup>Matt Richtel, “The Man Pushing America to Get on the Internet Faster,” *New York Times*, June 9, 2003, C1; Michael Totty, “Broadband: No Broad Consensus,” *Wall Street Journal*, January 14, 2002, R12.

**The model.** The model features  $S$  sectors, and in each of them there are two types of agents: producers, who use technologies, and innovators, who advance it. Within each sector, producers' choice of total factor productivity is endogenous. They can produce for free with a basic technology or license a more productive frontier technology from the sector's innovator and pay a quadratic adjustment cost. The return to adopting the frontier has two parts: a standalone return, which the sector earns if it adopts alone, and support from adoption in other sectors. Adoption choices are therefore strategic complements. At a given state of technology, adoption is a network coordination game, and I derive a sufficient statistic for the strength of its complementarities, the *Network Adoption Multiplier* (NAM). When the game has several equilibria, a public signal, which I call *hype*, coordinates sectors on the high-adoption equilibrium; without hype, sectors play the low-adoption equilibrium. Each innovator earns license income from adoption and invests in research. When research succeeds, the frontier advances and the old frontier becomes the sector's new basic technology; otherwise, the basic technology gradually catches up with the frontier. Research therefore widens the gap between the frontier and the basic technology, which raises the return to adoption, while adoption determines the license income that pays for research.

The first result characterizes when coordination can fail. The NAM is the spectral radius of the matrix of adoption complementarities: it is the factor by which an increase in adoption amplifies itself as it travels through the network. If  $NAM < 1$ , the amplification dies out and the adoption equilibrium is unique. Coordination failure requires  $NAM \geq 1$ . When every sector loses from adopting alone and mutual support covers these losses, high- and low-adoption equilibria coexist if and only if  $NAM > 1$ . Whenever the two equilibria differ, innovators earn more on the high-adoption equilibrium: the same frontier pays its innovator differently depending on how its users coordinate. Because research pays through future license income, persistent hype raises the value of innovation and, with it, research. Coordination over existing technology thereby determines the creation of future technology.

The feedback also runs in the opposite direction. A successful innovation moves the frontier ahead of the basic technology, strengthens adoption complementarities, and raises NAM: innovation can carry the economy from a unique adoption equilibrium into a state at which coordination can fail. Combining the two directions generates *macroeconomic hysteresis*. When hype is expected to stay absent, research stops, the basic technology catches up with the frontier, and the narrowing gap weakens adoption complementarities until the high- and low-adoption equilibria coincide. A later recovery in hype then has nothing to select: adoption revives only after new innovations have again moved the frontier ahead of the basic technology.

Coordination failures permanently lower productivity because they lower innovation. Adoption raises productivity at most up to the frontier, so switching between adoption equilibria changes the level of output and leaves its growth rate unchanged. Growth comes from advances of the frontier, and each advance is permanent: once it diffuses, it raises the productivity of the basic technology. By lowering research, a coordination failure slows frontier advancements, and the productivity lost persists after the failure ends.

**Measurement and identification.** In the model each sector has one technology, its frontier. In the data sectors use many technologies, so I take the model to the data one technology at a time. Each technology is a separate play of the adoption game, with the same network and standalone returns across technologies and a productivity gap specific to each. Sectors that use the same technologies benefit from each other’s adoption, so I measure the network from the overlap in sectors’ technology portfolios in U.S. job postings (Kalyani et al., 2025), observed before the estimation window. Estimating the strength of these links is harder, because linked sectors also adopt together when they face common shocks. I isolate shocks specific to one sector from changes in technology hiring at its largest firms (Gabaix and Koijen, 2024), and the network carries these shocks to the sectors that sector supports. Most sectors respond once support is large enough: they adopt a technology after the sectors that support them have adopted it. The estimates imply a NAM of 1.18, above the threshold of one that coordination failure requires.

**Quantification and Policy.** I calibrate a 278-sector version of the model to U.S. data – including the previously estimated network. The standalone return is negative in 261 of 278 sectors: in almost every sector, adoption pays only once the sectors that support it adopt. Coordination failure is frequent and costly for growth. I simulate the calibrated economy over a long horizon. In an average year, 52 percent of technologies have both a high- and a low-adoption equilibrium. Selecting the high-adoption equilibrium in every period raises annual TFP growth by 0.39 percentage points. Selecting the low-adoption equilibrium lowers it by 0.60 percentage points, which eliminates almost all growth. The cost runs through innovation. On the low-adoption equilibrium few sectors adopt, innovators earn little, and research falls to one fifteenth of its level on the high-adoption equilibrium. Relative to an economy with unit standalone returns and zero adoption complementarities, innovation contributes an additional 0.30 percentage points, half of TFP growth.

Next, I proceed analyzing three policies: a *hype* policy, an adoption subsidy, and a R&D subsidy. First, a *hype* policy is the costless benchmark that allows producers to coordinate on the highest adoption equilibrium every period. It delivers an increase of 0.39 in TFP growth, and serves as the increase subsidies target. Second, a targeted adoption subsidy to the top 10 sectors that have low standalone loss, and that provide high complementarities. Interestingly, six of the ten produce and run digital technology: computers, semiconductors, communications equipment, data processing and hosting, software, and internet services. These sectors lose little from adopting alone, so a modest subsidy makes them adopt, and their adoption supports many other sectors. Third, a uniform R&D subsidy delivers the same TFP gain at a significant higher fiscal cost. The reason is that, unlike the previous cases, it raises innovation on each equilibrium and leaves equilibrium selection to hype.

**Related literature.** The paper contributes to three literatures. First, it contributes to work on coordination failures in technology adoption. The big-push and technology-adoption literatures

study how complementarities and barriers determine whether available technologies are put to use (Rosenstein-Rodan, 1943; Murphy et al., 1989; Buera et al., 2021; Comin and Hobijn, 2010; Comin and Mestieri, 2018). For example, Alvarez et al. (2023) study strategic complementarities in the adoption of digital payments, where the incentive to adopt depends on others' adoption. In this paper, ex-post coordination in adoption determines the market for the innovation, therefore new technologies invented. Furthermore, the two-way feedback relates to models in which an endogenous state changes the scope for future coordination. In Schaal and Taschereau-Dumouchel (2025), coordination on a high technology raises capital accumulation, which in turn facilitates subsequent coordination. Here the endogenous state is technological: innovation strengthens complementarities in adoption, while the adoption equilibrium determines the return to further innovation. Implementation-cycle models generate endogenous fluctuations through the timing of technological adoption (Shleifer, 1986; Francois and Lloyd-Ellis, 2003). Here coordination failures shape the rate of innovation, and hence long-term growth.

Second, the paper contributes to the large literature of endogeneous growth pioneered by, among others, Romer (1990) and Aghion and Howitt (1992). I contribute to this literature showing how growth can be directly affected by failures in adopting technologies already invented, thereby discouraging future innovation, and hence growth itself. More specifically, a closely related work studying the interaction between adoption and innovation is Anzoategui et al. (2019). They show that both margins contribute to the productivity slowdown post Great Recession. Fernald (2015) and Fernald et al. (2017) argue that a reading of the post-2004 productivity slowdown is “bad luck”, an exogenous slowdown in the pace of innovation that precedes the Great Recession. In this paper, a persistent absence of hype produces a slowdown in the frontier that looks like “bad luck”, but is the endogenous result of a failure to coordinate on adoption. Conceptually, the feedback in my paper is closer to the seminal work Bresnahan and Trajtenberg (1995). They study a model with a general-purpose technology. There, innovation in the GPT and its applications are mutually reinforcing. Aghion et al. (2025) study innovation along a supply chain in which the adoption of a new technology in one sector depends on technological progress upstream; these dynamic complementarities can make the economy converge to different steady states.

Third, the paper contributes to the literature on networks (Acemoglu et al., 2012; Baqaee and Farhi, 2019, 2020; Liu, 2019). Kleinman et al. (2023) apply spectral analysis in a dynamic special network model, and Liu and Tsyvinski (2024) in a dynamic production network where producers face an adjustment cost of changing inputs. Although the conditions on the radius for uniqueness are similar, most of my paper focuses on connecting the spectral properties to multiplicity. Carvalho and Voigtländer (2014) show that input–output proximity predicts input adoption, and Liu and Ma (2024) study how knowledge spillovers across sectors shape innovation and the allocation of R&D. The network here carries a different interaction: adoption in one sector raises the return to using the frontier in another. This interaction creates a coordination problem in technology use, and innovation changes the strength of that coordination problem.

**Outline.** Section 2 presents the environment and defines equilibrium. Section 3 characterizes the coordination game and the Network Adoption Multiplier. Section 4 studies innovation incentives and the feedback between adoption and innovation. Section 5 derives the implications for long-run growth. Section 6 measures the adoption network and estimates its strength. Section 7 quantifies the growth consequences of coordination and evaluates counterfactual policies. Section 8 concludes.

## 2 Environment

Time is discrete and infinite,  $t = 0, 1, 2, \dots$ , with discount factor  $\beta \in (0, 1)$ . The economy has a representative household and a competitive final-good producer that aggregates goods from  $S$  sectors indexed by  $s \in \{1, \dots, S\}$ . Each sector contains a unit measure of producers indexed by  $i \in [0, 1]$ , a competitive assembler, and one monopolist innovator. There exists a public sunspot which takes value  $z_t \in \{0, 1\}$ . I call  $z_t = 1$  *hype*.

I introduce these agents in turn, then describe the timing and define the equilibrium.

**Household and final demand.** The household owns all producers and innovators, consumes the final good, and supplies production  $L_{st}$  and research labor  $L_{st}^R$ . Its preferences are

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t \left[ \log C_t - \sum_s L_{st}^R \right] \quad (1)$$

The first term is the utility of consuming the final good  $C_t$ . The second is the disutility of research labor  $L_{st}^R \geq 0$  supplied to sector  $s$ , whose marginal disutility is normalized to one, and the sum runs over sectors because research is sector-specific. The final good is

$$C_t = \prod_{s=1}^S \left( \frac{c_{st}}{b_s} \right)^{b_s} \quad (2)$$

with  $c_{st}$  consumption of sector  $s$ 's good,  $b_s > 0$ , and  $\sum_s b_s = 1$ . Production labor  $L_{st}$  is segmented, supplied inelastically at wage  $w_{st}$ , and it is in fixed supply such that  $L_{st} = 1$ . The competitive final-good producer turns sectoral goods into consumption according to (2), and the final good is the numeraire,  $\prod_s P_{st}^{b_s} = 1$ , where  $P_{st}$  is the price of sector  $s$ 's good. The household's income is labor income plus the profits of producers and innovators; payments between producers and innovators net to zero within it.

**Assemblers and producers.** In each sector a competitive assembler bundles the varieties  $y_{ist}$  supplied by producers into the sector good,

$$Y_{st} = \left( \int_0^1 y_{ist}^{1/\mu_s} di \right)^{\mu_s}, \quad \mu_s > 1, \quad (3)$$

so each variety faces demand

$$y_{ist} = \left( \frac{p_{ist}}{P_{st}} \right)^{-\frac{\mu_s}{\mu_s-1}} Y_{st}, \quad (4)$$

where  $P_{st} = \left( \int_0^1 p_{ist}^{-1/(\mu_s-1)} di \right)^{-(\mu_s-1)}$ .

Producer  $i$  in sector  $s$  has production function

$$y_{ist} = A_{ist}^{1-\alpha} \left( L_{ist}^{\ell_s} \prod_{s'=1}^S M_{s'ist}^{m_{s's}} \right)^\alpha, \quad (5)$$

where  $\alpha \in (0, 1)$ ,  $A_{ist} > 0$  is effective productivity,  $L_{ist}$  is production labor, and  $M_{s'ist}$  is good  $s'$  used as an intermediate input. Input shares satisfy  $\ell_s > 0$ ,  $m_{s's} \geq 0$ , and  $\ell_s + \sum_{s'} m_{s's} = 1$ .

**Productivity.** Two productivity levels are available in each sector. The basic productivity  $A_{st}^B > 0$  is freely available to every producer. The frontier productivity  $A_{st}^F \geq A_{st}^B$  is owned by the sector's single innovator, who innovates to advance it. The ratio  $q_{st} \equiv A_{st}^F / A_{st}^B \geq 1$  is the productivity gap between the frontier and basic productivity. The gap moves for two reasons. If the innovator's research succeeds, the frontier advances and the old frontier becomes the new basic productivity; otherwise, basic productivity diffuses toward the frontier.<sup>3</sup>

Producer  $i$  in sector  $s$  uses a share  $x_{ist} \in [0, 1]$  of the frontier. Let  $x_{st} \equiv \int_0^1 x_{ist} di$  denote sector  $s$  aggregate adoption and  $\mathbf{x}_{-st}$  collect the other sectors' aggregates. The producer's effective productivity is

$$A_{ist}(x_{ist}; \mathbf{x}_{-st}, A_{st}^B, A_{st}^F) = A_{st}^B + x_{ist} (A_{st}^F - A_{st}^B) \left( \kappa_s + \sum_{s' \neq s} \theta_{ss'} x_{s't} \right) - \frac{\chi_s}{2} A_{st}^B x_{ist}^2. \quad (6)$$

The term  $x_{ist}(A_{st}^F - A_{st}^B)$  is the part of the productivity gap the producer puts to use: at  $x_{ist} = 1$  the entire gap is in play, while at  $x_{ist} = 0$  none of it is. The term in brackets is the gross network return to using it. The *standalone return*  $\kappa_s \in \mathbb{R}$  is the return when no other sector adopts. It can be negative, in which case the frontier alone is worth less than the basic productivity it replaces because it requires complements that are not yet in use. The term  $\theta_{ss'} x_{s't}$ , which I call *network support*, captures what sector  $s'$  adoption adds to that return. In particular,  $\theta_{ss'} \geq 0$  measures how much sector  $s'$  adoption raises the productivity that sector  $s$  obtains from using its frontier productivity. I assume  $\theta_{ss} = 0$ , so producers receive support only from other sectors and coordination occurs across sectors.<sup>4</sup> The final term is a convex adjustment cost governed by  $\chi_s > 0$ . It is paid in units of forgone basic productivity: reorganizing production around the frontier disrupts current production, and the cost rises more than proportionally with adoption.

<sup>3</sup>The catching up reflects imitation as in [Lucas and Moll \(2014\)](#) and [Benhabib et al. \(2021\)](#). The full law of motion is specified in (14).

<sup>4</sup>Appendix C.5 extends the environment to  $\theta_{ss} > 0$ . The expressions are less clean, and the results continue to hold under an assumption on primitives.

Factoring out  $A_{st}^B$ , the two productivity levels enter only through the gap:

$$\frac{A_{ist}}{A_{st}^B} = 1 + x_{ist}(q_{st} - 1) \left[ \kappa_s + \sum_{s' \neq s} \theta_{ss'} x_{s't} \right] - \frac{\chi_s}{2} x_{ist}^2. \quad (7)$$

The first term is the basic productivity, which the producer retains regardless of adoption. The second is the gain from adoption: the share adopted,  $x_{ist}$ , times the productivity available to be adopted,  $q_{st} - 1$ , times the gross network return. The last term is the adjustment cost, now expressed in units of basic productivity.

**Innovation payoff.** Given productivity  $A$ , sector wage  $w_{st}$ , and sector prices  $\mathbf{P}_t = (P_{1t}, \dots, P_{St})$ , producer  $i$ 's operating profit before license payments is

$$\begin{aligned} \Pi_{ist}^Y(A, \mathbf{P}_t, w_{st}) &= \max_{p_{ist}, y_{ist}, L_{ist}, \mathbf{M}_{ist}} \left\{ p_{ist} y_{ist} - w_{st} L_{ist} - \sum_{s'} P_{s't} M_{s'ist} \right\} \\ \text{s.t. } & \text{(5) and (4).} \end{aligned} \quad (8)$$

Frontier use requires a license from the innovator. Producer  $i$  and innovator  $s$  bargain jointly over adoption  $x_{ist}$  and the license fee  $\phi_{ist}$ , taking  $\mathbf{x}_{-st}$ ,  $\mathbf{P}_t$ , and  $w_{st}$  as given:

$$(x_{ist}^*, \phi_{ist}) \in \arg \max_{x \in [0,1], \phi \geq 0} \phi^{\zeta_s} \left[ \Pi_{ist}^Y(A_{ist}(x; \mathbf{x}_{-st}, A_{st}^B, A_{st}^F), \mathbf{P}_t, w_{st}) - \Pi_{ist}^Y(A_{st}^B, \mathbf{P}_t, w_{st}) - \phi \right]^{1-\zeta_s}, \quad (9)$$

where  $\zeta_s \in (0, 1)$  is the innovator's bargaining share. The outside option is producing with the basic productivity. Because each pair is atomistic, it takes other sectors' aggregate adoption as given. The license lasts one period and is agreed before production, so future innovation lies outside the bargain.

**Hype and equilibrium selection.** Throughout I focus on producers' symmetric strategies, hence  $x_{ist} = x_{st}$ , so sector  $s$  aggregate adoption is the common choice of its producers. Each sector's choice depends on adoption in the other sectors through the support term. Stacking these choices,

$$\mathbf{x}(\mathbf{q}_t) = \begin{pmatrix} x_{1t}^*(\mathbf{x}_{-1t}; q_{1t}) \\ \vdots \\ x_{St}^*(\mathbf{x}_{-St}; q_{St}) \end{pmatrix}, \quad (10)$$

where row  $s$  is the adoption  $x_{st}^*$  that sector  $s$ 's pairs bargain to, given the other sectors' aggregate adoption  $\mathbf{x}_{-st}$  and its own productivity gap  $q_{st}$ .

Several fixed points of (10) can arise because adoption is complementary across sectors. Section 3 shows that the equilibrium set has a least and a greatest element (Lemma 1), denoted  $\underline{\mathbf{x}}(\mathbf{q}_t) \leq \bar{\mathbf{x}}(\mathbf{q}_t)$ . They coincide when the adoption equilibrium is unique and differ when the extremal equilibria are distinct. A public sunspot  $z_t \in \{0, 1\}$  selects between them and follows a

Markov chain with transition matrix  $\Pi = [\pi_{zz'}]$ :

$$\mathbf{x}(\mathbf{q}_t, z_t) = \begin{cases} \bar{\mathbf{x}}(\mathbf{q}_t) & \text{hype, } z_t = 1, \\ \underline{\mathbf{x}}(\mathbf{q}_t) & \text{no hype, } z_t = 0. \end{cases} \quad (11)$$

I call  $z_t = 1$  *hype*: sectors expect complementary sectors to adopt, and these expectations coordinate them on the greatest equilibrium. Without hype, they expect complementary sectors to hold back and coordinate on the least equilibrium.<sup>5</sup> Hype therefore matters only when the same fundamentals support different adoption outcomes.

The selected adoption profile determines the license fee:

$$\phi_{st}(\mathbf{A}_t^B, \mathbf{q}_t, z_t) = \begin{cases} \phi_{ist}(\bar{\mathbf{x}}_{-st}(\mathbf{q}_t); A_{st}^B, q_{st} A_{st}^B) & \text{hype, } z_t = 1, \\ \phi_{ist}(\underline{\mathbf{x}}_{-st}(\mathbf{q}_t); A_{st}^B, q_{st} A_{st}^B) & \text{no hype, } z_t = 0. \end{cases} \quad (12)$$

The fee is the outcome of (9) evaluated at the adoption profile selected by hype. Since producers are symmetric, the innovator's license income is  $\int_0^1 \phi_{ist} di = \phi_{st}$ . Thus the same frontier, at the same basic productivity, can generate different license income depending on which adoption equilibrium sectors coordinate on. This is the channel through which coordination over adoption affects the return to innovation.

**Innovators.** Innovator  $s$  collects the license income and chooses the research labor  $L_{st}^R$  to hire at wage  $w_{st}^R$ . Research intensity is produced from labor,  $r_{st} = (C_t / P_{st} Y_{st}) L_{st}^R$ : a unit of intensity requires  $P_{st} Y_{st} / C_t$  units of research labor, so research in a large sector requires proportionally more labor than research in a small one, and research effort is measured in units comparable to license income.

Success occurs with probability  $h_s(r_{st})$ , where  $h_s : \mathbb{R}_+ \rightarrow [0, 1]$  is continuous and nondecreasing, continuously differentiable, strictly increasing and strictly concave wherever  $h_s(r) < 1$ , and  $h_s(0) = 0$ . Thus no research means no chance of success, and additional research raises the chance of success at a decreasing rate.

The innovator values license income net of the research bill, discounted with the household's stochastic discount factor  $\beta C_t / C_{t+1}$ , since the household owns it. Its value at the state  $(\mathbf{A}_t^B, \mathbf{q}_t, z_t)$  is

$$V_s(\mathbf{A}_t^B, \mathbf{q}_t, z_t) = \phi_{st}(\mathbf{A}_t^B, \mathbf{q}_t, z_t) + \max_{L_{st}^R \geq 0} \left\{ -w_{st}^R L_{st}^R + \mathbb{E}_t \left[ \beta \frac{C_t}{C_{t+1}} V_s(\mathbf{A}_{t+1}^B, \mathbf{q}_{t+1}, z_{t+1}) \right] \right\}. \quad (13)$$

The first term is the license income (12). It is predetermined given the state: the fee was bargained on today's productivities and today's hype, and research this period cannot change it, so it lies outside the maximization. The second term is forward-looking. The innovator chooses research labor, paying  $w_{st}^R L_{st}^R$  today against the expected continuation value, discounted with

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<sup>5</sup>The two equilibria need not be distinct.

the household's stochastic discount factor. The expectation is taken over the exogenous hype process, and the expected productivity induced by other sectors' research. In particular, each sector's productivity law of motion reads:

$$\begin{cases} A_{s,t+1}^F = \gamma_s A_{st}^F & \text{w.p. } h_s(r_{st}), \\ A_{s,t+1}^F = A_{st}^F & \text{w.p. } 1 - h_s(r_{st}), \end{cases} \quad \begin{cases} A_{s,t+1}^B = A_{st}^F & \text{w.p. } h_s(r_{st}), \\ A_{s,t+1}^B = (A_{st}^B)^{\delta_s} (A_{st}^F)^{1-\delta_s} & \text{w.p. } 1 - h_s(r_{st}), \end{cases} \quad (14)$$

with  $\gamma_s > 1$  and  $\delta_s \in [0, 1)$ . The left block gives the frontier and the right block the basic productivity. On success, the old frontier becomes the new basic productivity and the frontier advances by  $\gamma_s$ : the innovation step is fixed, and research determines the probability that it is taken. On failure, the frontier is unchanged and basic productivity diffuses toward it. Next period's basic productivity is a geometric average of today's basic and frontier productivities, with weight  $\delta_s$  on the basic. Thus  $\delta_s$  close to one implies slow diffusion, while  $\delta_s = 0$  implies that the basic catches up with the frontier in one period.

Equivalently, dividing (14) by  $A_{st}^B$ , the productivity gap evolves according to

$$q_{s,t+1} = \begin{cases} \gamma_s, & \text{with probability } h_s(r_{st}), \\ q_{st}^{\delta_s}, & \text{with probability } 1 - h_s(r_{st}). \end{cases} \quad (15)$$

The timing is as follows. At the beginning of period  $t$ , agents observe  $(\mathbf{A}_t^B, \mathbf{q}_t, z_t)$ . Producer–innovator pairs bargain over frontier adoption and license fees; producers hire inputs and produce; fees are settled; innovators choose research; and innovation outcomes are realized. The state then becomes  $(\mathbf{A}_{t+1}^B, \mathbf{q}_{t+1}, z_{t+1})$ .

**Definition 1.** A sunspot equilibrium is a collection of functions for licenses  $\{x_{ist}, \phi_{ist}\}$ , research and values  $\{r_{st}, V_s\}$ , prices  $\{p_{ist}, P_{st}, w_{st}, w_{st}^R\}$ , allocations  $\{y_{ist}, Y_{st}, L_{ist}, M_{s'ist}, L_{st}^R, c_{st}, C_t\}$  such that, at every state  $(\mathbf{A}_t^B, \mathbf{q}_t, z_t)$ , including off-path states:

- (i) Optimization. Producers solve (8) subject to (5) and (4); each producer–innovator pair solves (9), taking  $\mathbf{x}_{-st}$ , prices, and wages as given; the household and final-good producer optimize; and each innovator solves (13) subject to (14), taking the other sectors' policies, the hype selection rule, and  $\Pi$  as given, with  $V_s$  the resulting value.
- (ii) Hype selection. Producers within each sector choose symmetrically,  $x_{ist} = x_{st}$ , and  $z_t$  selects a fixed point of (10) according to (11): the greatest equilibrium under hype and the least without it.
- (iii) Market clearing. Production labor clears in every sector,  $\int_0^1 L_{ist} di = 1$ ; the household supplies the research labor  $L_{st}^R$  that innovator  $s$  hires; and, for every sector  $s'$ ,

$$Y_{s't} = c_{s't} + \sum_{s=1}^S \int_0^1 M_{s'ist} di.$$

License payments equal the bargained fees and cancel within the household.

**Discussion.** A few comments are in order regarding the modeling assumptions. First, I assume that the adoption quadratic cost in (6) is scaled in units of foregone basic productivity  $A_{st}^B$ . One might think of at least three alternatives: the frontier level; the difference between  $A_{st}^F$  and  $A_{st}^B$ ; foregone input. The intuition behind the baseline is that retraining workers, reorganizing production, and change technology divert output produced with the basic technology. The foregone-input alternative says the same thing in a different unit: the cost is a wage bill,  $w_{st} \frac{\chi_s}{2} x_{ist}^2$ . As I will show, in equilibrium the wage is proportional to  $A_{st}^B$ , therefore it coincides with the baseline up to the sector's labor share. The other two alternatives tie the cost to a technology the producer hasn't adopted yet. With the frontier level,  $\frac{\chi_s}{2} A_{st}^F x_{ist}^2$ , adoption would become more expensive with every success in the sector. With the difference,  $\frac{\chi_s}{2} (A_{st}^F - A_{st}^B) x_{ist}^2$ , cost and return would scale by the same factor, the gap would drop out of the best response, and the state of technology would play no role in the coordination game. Both would require the producer to pay more for the same reorganization because the frontier it has yet to adopt is more productive, which is hard to justify.

Second, I assume that  $\theta_{ss} = 0$ . Multiplicity through within-sector complementarity has been widely discussed in the literature (Katz and Shapiro, 1985; Farrell and Saloner, 1985), so I extend the model to  $\theta_{ss} \geq 0$  and show that the results go through under a mild assumption on the shifter of the quadratic cost. Intuitively, this is because the innovator bargains efficiently with the producer, and therefore understands that a higher level of adoption would increase the surplus. Since it is optimal for the producers as well, they are willing to accept that contract. For details, see Appendix C.5.

Third, the production function for research is  $r_{st} = (C_t / P_{st} Y_{st}) L_{st}^R$ , or alternatively, using the revenue-based Domar weight,  $r_{st} = L_{st}^R / \lambda_s$ . This means that the higher the revenue share, the lower the research output of a worker. This assumption is in the tradition of growth models such as Klette and Kortum (2004) and Akcigit and Kerr (2018), where the cost scales with the number of product lines. This assumption allows the size of a sector to be independent of the R&D choice. Relaxing it and imposing  $r_{st} = L_{st}^R$  would give a persistent, but constant, gap in research across sectors. Thus, neither the economics nor the results would be affected.

Fourth, labor markets are segmented: production labor is supplied inelastically to each sector, and research labor to each sector at a linear disutility, so workers move neither between production and research nor across sectors. This shuts down the reallocation channel of the growth literature, in which innovation draws workers out of production and out of other sectors. The payoff is that coordination affects output through productivity alone, and each innovator's research responds to its own license income at a wage,  $w_{st}^R = C_t$ , that its choices leave unchanged. With a common labor market, research in one sector would raise wages everywhere, so coordination would raise research by less and part of its gain would go to wages; the coordination game would be the same and the numbers in Section 7 smaller.

The analysis proceeds as follows. Section 3 solves the coordination game and the bargain at a given state. Section 4 takes the fee as the innovator's payoff and solves the research problem.

Section 5 aggregates the resulting productivity dynamics into the growth rate of the economy.

### 3 The Coordination Game

This section characterizes adoption at a given gap vector. A producer that adopts the frontier today carries no state into tomorrow: basic productivity follows (14) whatever the producer chooses, and today's adoption enters the current license fee alone. Adoption is therefore a static game played each period at the current gap vector  $\mathbf{q}$ . I fix  $\mathbf{q}$ , suppress time subscripts, and characterize the equilibria of this game (Section 3.1); I then show how the gaps, which innovation sets, determine whether coordination can fail (Section 3.3), and what the equilibrium played pays the innovator (Section 3.4). These two links—from innovation to coordination and from coordination to innovation—are the feedback Section 4 closes.

#### 3.1 Adoption equilibria

Pair  $s$  takes the other sectors' adoption  $\mathbf{x}_{-s}$  as given. Its operating-profit gain from the frontier is strictly increasing in  $A_s/A_s^B$  (Section 3.4), so the pair chooses  $x_s \in [0, 1]$  to maximize the right-hand side of (7),

$$x_s(q_s - 1) \left( \kappa_s + \sum_{s' \neq s} \theta_{ss'} x_{s'} \right) - \frac{\chi_s}{2} x_s^2.$$

The objective is strictly concave in  $x_s$ , so the best response is unique:

$$\text{BR}_s(\mathbf{x}_{-s}, q_s) = \max \left\{ \min \left\{ \hat{\kappa}_s(q_s) + \sum_{s' \neq s} \hat{\theta}_{ss'}(q_s) x_{s'}, 1 \right\}, 0 \right\}, \quad (16)$$

where

$$\hat{\kappa}_s(q_s) = \frac{q_s - 1}{\chi_s} \kappa_s, \quad \hat{\theta}_{ss'}(q_s) = \frac{q_s - 1}{\chi_s} \theta_{ss'}$$

are the effective standalone return and the effective spillover from  $s'$ , collected in the vector  $\hat{\kappa}(\mathbf{q})$  and the effective network  $\hat{\Theta}(\mathbf{q})$ : the primitives  $\kappa_s$  and  $\theta_{ss'}$ , scaled by the productivity gap per unit of adjustment cost. Away from the bounds,  $\hat{\kappa}_s$  is the intercept of the best response and  $\hat{\theta}_{ss'}$  its slope on  $x_{s'}$ . Notice here that, while  $\theta_{ss'}$  is a primitive, the effective spillover depends on the gap  $q_s$ . This is the sense in which the network is endogenous. Namely, as the frontier advances or basic productivity catches up with it, the gap changes, and with it how strongly a sector's adoption responds to adoption elsewhere. At  $q_s = 1$ , both vanish and sector  $s$  does not adopt, regardless of what the other sectors do.

Since the slopes are nondecreasing in the other sectors' adoption, choices are strategic complements, therefore Tarski's fixed-point theorem applies. It follows that the set of equilibria is nonempty and forms a complete lattice, with well defined highest and lowest equilibria. Nonetheless, the two may coincide. The following lemma states this formally.

**Lemma 1** (Existence). *For every  $\mathbf{q}$ , an adoption equilibrium exists, and there are a least and a greatest equilibrium,*

$$\underline{\mathbf{x}}(\mathbf{q}) \leq \bar{\mathbf{x}}(\mathbf{q}).$$

*Best-response iteration from no adoption converges to the least equilibrium, and iteration from full adoption converges to the greatest.*

These extremal equilibria are the two profiles selected by hype. Without hype, sectors coordinate on the least equilibrium, each expecting the others not to adopt; under hype, they coordinate on the greatest, each expecting the others to adopt. The effective standalone returns locate these equilibria. A sector with  $\hat{\kappa}_s > 0$  adopts even when no other sector does; I call it a *seed*. A sector with  $\hat{\kappa}_s \leq 0$  requires support from other sectors to adopt. If  $\hat{\kappa}_s + \sum_{s'} \hat{\theta}_{ss'} \leq 0$ , it does not adopt even when every other sector adopts fully. Thus the intercepts and row sums of the effective network determine where the extremal equilibria lie. The following lemma formalizes the cases.

**Lemma 2.** *Fix  $\mathbf{q}$ .*

- (i)  $\underline{\mathbf{x}} = \mathbf{0}$  if and only if  $\hat{\kappa} \leq \mathbf{0}$ . Otherwise every seed is active on the least equilibrium.
- (ii)  $\bar{\mathbf{x}} = \mathbf{1}$  if and only if  $\hat{\kappa} + \hat{\Theta}\mathbf{1} \geq \mathbf{1}$ . Otherwise some sector adopts less than fully on the greatest equilibrium.
- (iii) If  $\hat{\kappa} \geq \mathbf{1}$ , full adoption is the unique equilibrium.

The least equilibrium is no adoption exactly when no sector gains from adopting alone. The greatest is full adoption exactly when every sector wants to use its entire frontier when the others do; between these cases the extremal equilibria can be partial. Part (iii) is the case with nothing to coordinate: every sector adopts fully on its own, so expectations play no role.

The next subsection characterizes when the effective network can sustain such an equilibrium.

### 3.2 The Network Adoption Multiplier

Whether network support can sustain a distinct adoption equilibrium depends on the slopes of sectors' best responses, collected in  $\hat{\Theta}(\mathbf{q})$ . I show that a single summary statistic measures the strength of feedback through the effective network.

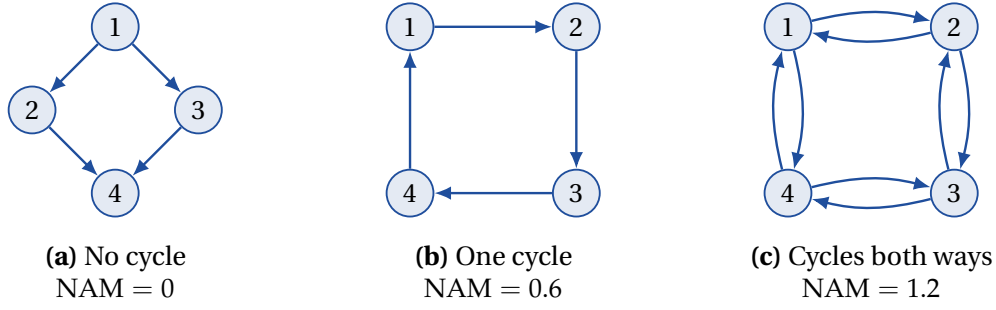
**Definition 2.** *The Network Adoption Multiplier at  $\mathbf{q}$  is*

$$\text{NAM}(\mathbf{q}) \equiv \rho(\hat{\Theta}(\mathbf{q})),$$

*where  $\rho(\cdot)$  is the spectral radius, i.e., the largest eigenvalue associated to  $\hat{\Theta}(\mathbf{q})$ .*

To illustrate, consider a group of sectors connected with each other through complementarities. As each sector receives more network support, it raises its own adoption, which in turn provides support to other sectors. The NAM measures whether the feedbacks of mutual support vanish or reinforce in each round. The magnitude depends on the productivity gaps  $\mathbf{q}$  and on the primitive links  $\Theta$ .

**Figure 1: THREE NETWORKS AND THEIR MULTIPLIERS**



Notes: Four sectors; an arrow from  $s'$  to  $s$  means that sector  $s'$ 's adoption supports sector  $s$ ,  $\theta_{ss'} > 0$ , and every drawn link has the same effective spillover  $\hat{\theta}_{ss'} = 0.6$ . Panel (a) has no directed cycle, so the multiplier is zero. Panel (b) closes one cycle and has NAM = 0.6. Panel (c) runs every link both ways and has NAM = 1.2.

To provide intuition, Figure 1 illustrates the multiplier in three networks of four sectors. Their effective networks are

$$\hat{\Theta}_{(a)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.6 & 0.6 & 0 \end{pmatrix}, \quad \hat{\Theta}_{(b)} = \begin{pmatrix} 0 & 0 & 0 & 0.6 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \end{pmatrix}, \quad \hat{\Theta}_{(c)} = \begin{pmatrix} 0 & 0.6 & 0 & 0.6 \\ 0.6 & 0 & 0.6 & 0 \\ 0 & 0.6 & 0 & 0.6 \\ 0.6 & 0 & 0.6 & 0 \end{pmatrix}.$$

In panel (a), sector 1 supports sectors 2 and 3, which support sector 4, so an increase in sector 1's adoption travels down to sector 4 and induces no round of feedback, hence NAM = 0. In panel (b), the links form one cycle, so an increase in adoption returns to the sector it started from, shrunk by 0.6 at each link, and NAM = 0.6. In panel (c), every link runs both ways, so support returns through several paths, the rounds reinforce, and NAM = 1.2.

When NAM < 1, an increase in adoption raises adoption elsewhere, but each round of feedback is weaker than the one before it. Mutual support therefore cannot sustain a second equilibrium. In panel (b) of Figure 1, for example, the effective spillover on each link is 0.6, so a unit of adoption returns to its origin as  $0.6^4 \approx 0.13$  after traveling once around the cycle. The least and greatest equilibria consequently coincide.

The first result provides a sufficient condition on the NAM to have uniqueness.

**Theorem 1 (Uniqueness).** *Fix  $\mathbf{q}$ . If  $\text{NAM}(\mathbf{q}) < 1$ , the adoption equilibrium is unique, and best-response iteration converges to it from any initial profile.*

When NAM ≥ 1, mutual support need not fade. Whether it sustains a distinct equilibrium then also depends on the standalone returns. Let  $\mathbf{c} \geq \mathbf{0}$  be an eigenvector of  $\hat{\Theta}(\mathbf{q})$  associated with NAM( $\mathbf{q}$ ),

$$\hat{\Theta}(\mathbf{q})\mathbf{c} = \text{NAM}(\mathbf{q})\mathbf{c},$$

normalized so that  $\max_s c_s = 1$ . Such an eigenvector always exists, and some of its entries can be

zero.<sup>6</sup> Along this adoption profile, the support received by sector  $s$  is  $\text{NAM}(\mathbf{q})\mathbf{c}_s$ . Define

$$\underline{\kappa}_s(\mathbf{q}) \equiv -(\text{NAM}(\mathbf{q}) - 1)\mathbf{c}_s. \quad (17)$$

The threshold  $\underline{\kappa}_s(\mathbf{q})$  is the largest loss from adopting alone that mutual support along  $\mathbf{c}$  can cover in sector  $s$ . Of the support  $\text{NAM}(\mathbf{q})\mathbf{c}_s$  that reaches the sector,  $\mathbf{c}_s$  is needed to sustain its own adoption and  $(\text{NAM}(\mathbf{q}) - 1)\mathbf{c}_s$  can offset a negative standalone return. This surplus is positive exactly when the multiplier exceeds one.

**Theorem 2** (Multiplicity). *Fix  $\mathbf{q}$ .*

- (i) *(sufficiency) If  $\hat{\kappa}_s(q_s) \in [\underline{\kappa}_s(q_s), 0)$  for every  $s$  with  $\mathbf{c}_s > 0$ , and  $\text{NAM}(q_s) > 1$ . Then  $\underline{\mathbf{x}}(\mathbf{q}) = \mathbf{0}$ , and  $\underline{\mathbf{x}}(\mathbf{q}) \neq \bar{\mathbf{x}}(\mathbf{q}) \geq \mathbf{c}_s$*
- (ii) *(necessity) If there is multiplicity, i.e.,  $\underline{\mathbf{x}}(\mathbf{q}) \neq \bar{\mathbf{x}}(\mathbf{q})$  then  $\text{NAM}(\mathbf{q}) \geq 1$ .*
- (iii) *(fee) If  $\underline{\mathbf{x}}(\mathbf{q}) \neq \bar{\mathbf{x}}(\mathbf{q})$ , every innovator is paid weakly more on the greatest equilibrium,*

$$\phi_s(\bar{\mathbf{x}}_{-s}; \cdot) \geq \phi_s(\underline{\mathbf{x}}_{-s}; \cdot),$$

*strictly in every sector whose adoption differs across the two.*

Every sector loses from adopting alone, so no adoption is an equilibrium. When  $\text{NAM} > 1$ , mutual support generates a surplus along  $\mathbf{c}$ ; if that surplus covers every standalone loss, a positive equilibrium can also sustain itself. Coordination fails because adoption is unprofitable alone: each sector loses by moving first, while all gain when they move together.

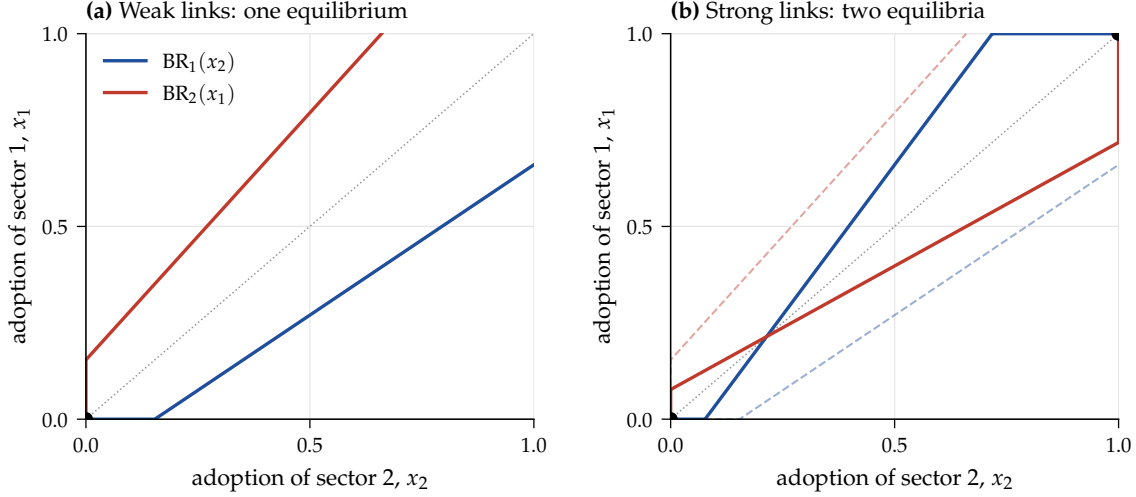
Part (ii) gives the general implication used throughout the paper: coordination failure requires  $\text{NAM} \geq 1$ . A multiplier at or above one need not be sufficient. Sectors with positive standalone returns can raise the least equilibrium above zero, while sufficiently large standalone losses can prevent mutual support from sustaining a distinct greatest equilibrium. The multiplier governs the strength of feedback; standalone returns determine where sectors' best responses lie.

Panel (c) of Figure 1 gives a simple example. Its eigenvector  $\mathbf{c}$  is the vector of ones and  $\text{NAM} = 1.2$ , so mutual support can cover a standalone loss of 0.2 in every sector. With losses no larger than 0.2, no adoption and full adoption can both be equilibria. If the loss rises to 0.5 in two sectors, the same network no longer sustains full adoption. Thus the multiplier measures the strength of mutual support, while standalone returns determine whether that support is enough to generate coordination failure.

Importantly, the equilibrium played also determines what innovators are paid. On the high adoption equilibrium, the profits from using the frontier rather than the basic productivity are higher. Since the license fee is proportional to the profit gain, this will result in a weakly higher license fee. The same frontier can thus earn different license fees depending on how sectors coordinate, providing the link from the coordination game to the innovation problem.

<sup>6</sup>A nonnegative eigenvector associated with the spectral radius exists for every nonnegative matrix (Perron–Frobenius). When  $\hat{\Theta}(\mathbf{q})$  is reducible there can be several; the results below hold for each of them.

**Figure 2: TWO SECTORS: STRONGER SPILLOVERS**



*Notes:* In each panel the blue line is the best response of sector 1 to sector 2's adoption,  $BR_1(x_2)$ , and the red line the best response of sector 2 to sector 1's,  $BR_2(x_1)$ ; the dotted line is the diagonal and the dots mark the least and greatest equilibria. Panel (b) doubles both spillovers relative to panel (a), holding every other parameter fixed; the dashed lines repeat panel (a). For the parametrization used, see Appendix C.4.

Both theorems hold at a fixed gap vector. The next subsection varies the gaps and shows how innovation changes the strength of the coordination problem.

### 3.3 Innovation moves the coordination game

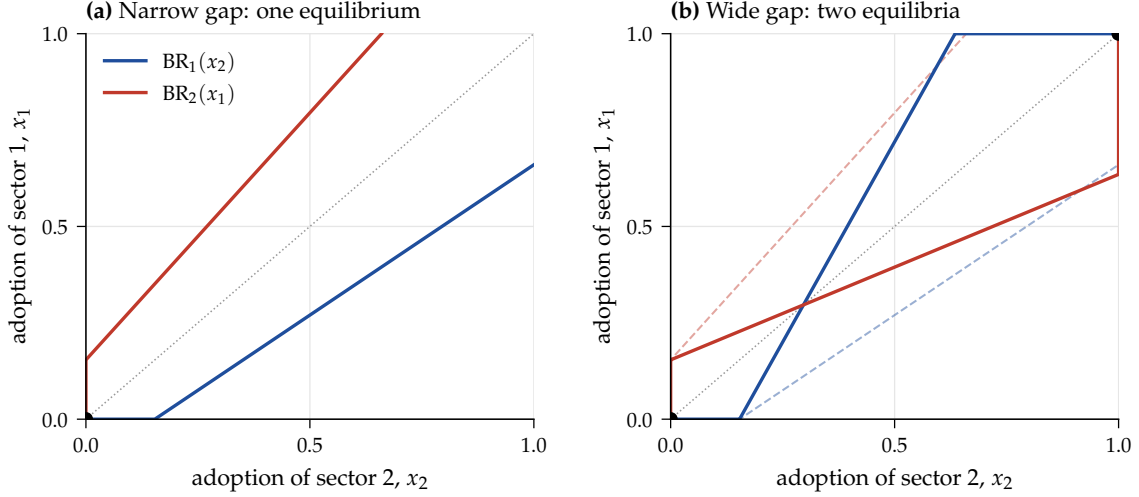
Two objects determine the effective network: the spillovers, which are primitives, and the productivity gaps, which innovation sets. Figure 2 varies the first and Figure 3 the second in a two-sector economy.<sup>7</sup>

**Stronger links.** Figure 2 plots each sector's best response to the other's adoption. In panel (a), both sectors would face a loss if adopting alone, so each best response remains at zero until adoption by the other sector provides enough support to cover that loss. Above that point, the best response rises with the other's adoption. The slopes are below one, and the unique intersection is no adoption.

Panel (b) doubles the spillovers and holds everything else fixed. The best responses become steeper and the adoption needed from the other sector before each starts adopting falls. Two further intersections appear, one interior and one at full adoption: stronger spillovers provide enough support to cover the same standalone losses as in panel (a). Accordingly, the multiplier rises from 0.78 to 1.56. In panel (b), each effective standalone return of  $-0.12$  lies above the threshold  $-0.56$  in (17), and the two equilibria characterized by Theorem 2 coexist.

<sup>7</sup>Appendix C.4 reports the parametrization.

**Figure 3: TWO SECTORS: WIDER GAPS**



*Notes:* Lines and markers as in Figure 2. Panel (a) is the economy of panel (a) of Figure 2; panel (b) widens the gap  $q$  in both sectors, holding every other parameter fixed; the dashed lines repeat panel (a). For the parametrization used, see Appendix C.4.

**Wider gaps.** Figure 3 instead holds the spillovers fixed and widens the productivity gaps. Panel (a) is the same economy as panel (a) of Figure 2. In panel (b), adopting the frontier puts more productivity to use, so the same support from the other sector has a larger effect on the return to adoption. Both best responses become steeper, and full adoption becomes an equilibrium alongside no adoption.

This is how innovation moves the coordination game. A success raises the productivity available for adoption, while failures allow basic productivity to diffuse toward the frontier. Research therefore helps determine whether the economy lies in the region of multiple equilibria, the set of gap vectors at which the least and greatest equilibria differ. The next subsection derives what the equilibrium played pays the innovator.

### 3.4 The license fee

The innovator is paid through the bargain (9). Sector revenue is a fixed multiple of consumption,  $R_{st} \equiv P_{st}Y_{st} = \lambda_s C_t$ , where  $\lambda$  contains the revenue shares implied by the input–output structure and final demand. Operating profit before the license payment is the share  $1 - \alpha/\mu_s$  of revenue and rises with productivity with elasticity  $(1 - \alpha)/(\mu_s - \alpha)$ . Since agreement and disagreement face the same prices and wage, the license fee is the innovator’s share of the producer’s operating-profit gain from using the frontier:

$$\phi_{st} = \zeta_s \left(1 - \frac{\alpha}{\mu_s}\right) R_{st} \left[1 - \left(\frac{A_{st}^B}{A_{st}}\right)^{\frac{1-\alpha}{\mu_s-\alpha}}\right], \quad (18)$$

where  $\zeta_s$  is the innovator's bargaining share,  $(1 - \alpha / \mu_s) R_{st}$  is profit at the productivity the producer uses, and the term in brackets is the share of that profit generated by using optimally the frontier as opposed to the basic productivity. It is worth highlighting that the fee is zero without adoption and weakly higher on the greatest equilibrium than on the least (Theorem 2). Thus the same frontier can make the innovator earn differently depending on how sectors coordinate. Finally, notice that the fee per unit of revenue depends on the state only through the gaps and hype. To see this divide by revenue:

$$f_s(\mathbf{q}_t, z_t) \equiv \frac{\phi_{st}}{R_{st}} = \zeta_s \left(1 - \frac{\alpha}{\mu_s}\right) \left[1 - \left(\frac{A_{st}^B}{A_{st}}\right)^{\frac{1-\alpha}{\mu_s-\alpha}}\right], \quad (19)$$

where  $A_{st}/A_{st}^B$  is (7) evaluated at the profile  $\mathbf{x}(\mathbf{q}_t, z_t)$  selected by hype in (11).

Research changes the productivity available for future adoption, while hype determines which adoption equilibrium is played. Section 4 combines these two forces in the innovator's research problem.

## 4 Innovation Incentives

This section closes the feedback between coordination and innovation. Section 3 characterized the coordination game for given productivity levels and showed how gaps affect the adoption and the license fees. I now turn the attention to the innovation process that determines those gaps. I first show that the innovator's value scales with sectoral revenue  $\lambda_s C_t$  so that research depends only on the gaps and on hype. I then show that hype raises research. When hype is expected to stay absent, research stops, diffusion narrows the gaps, and the region of multiple equilibria closes. When hype is present and research is active, successes widen the gaps and the region reopens.

### 4.1 The innovator's problem

The innovator's problem (13) is written in levels, on the state  $(\mathbf{A}_t^B, \mathbf{q}_t, z_t)$ , but every flow scales with sector revenues  $R_{st} = \lambda_s C_t$ . In particular, the fee is  $R_{st} f_s(\mathbf{q}_t, z_t)$  and the unit research cost is  $R_{st} r_{st}$ , therefore they both depend on the aggregate level of the economy only through the sectoral revenue. In addition, the household's discount factor  $\beta C_t / C_{t+1}$  offsets the growth of revenues between  $t$  and  $t + 1$ . As a consequence, the innovator's problem can be rescaled as the following lemma states formally.

**Lemma 3.** *The value of innovator  $s$  is proportional to its sector's revenue,*

$$V_s(\mathbf{A}_t^B, \mathbf{q}_t, z_t) = R_{st} v_s(\mathbf{q}_t, z_t),$$

where  $v_s$  is the unique bounded solution of

$$v_s(\mathbf{q}, z) = f_s(\mathbf{q}, z) + \max_{r \geq 0} \left\{ -r + \beta \mathbb{E} \left[ h_s(r) v_s((\gamma_s, \mathbf{q}'_{-s}), z') + (1 - h_s(r)) v_s((q_s^{\delta_s}, \mathbf{q}'_{-s}), z') \mid \mathbf{q}, z \right] \right\}, \quad (20)$$

The maximizer is unique and defines the research policy  $r_s^*(\mathbf{q}, z)$ .

Lemma 3 pins down one innovator's research given the others'. The other innovators enter (20) through their success probabilities. These determine the other sectors' gaps and hence the support the innovator's sector receives. A research equilibrium is a profile of success probabilities, one for each sector and state, at which every innovator's research is optimal given the others'. Each innovator's research is unique given the others'. Nonetheless, as in the adoption game, several profiles may be equilibria.

A success by another innovator widens the gap of its sector. This raises the support the innovator's own sector receives after its own success. Therefore greater research by the others can raise the innovator's success premium, i.e., the discounted gain in continuation value from a success relative to a failure. I assume that this complementarity holds at every state (Assumption B.1). Research choices are then strategic complements, as adoption choices are. Hence the set of research equilibria is a nonempty complete lattice with a least and a greatest element (Theorem B.1). Throughout I focus on the greatest research equilibrium.<sup>8</sup> Given the others' equilibrium research, each innovator's value is then the unique bounded solution of (20).

The next subsection studies how this equilibrium changes with hype, and Section 4.3 how research, in turn, moves the multiplier and the gap vector into and out of the region of multiple equilibria.

## 4.2 Hype raises research

Research is an investment. The innovator pays the research cost today, and a success pays through the fees of the following periods. Namely, a wider gap increases the profit gain from using the frontier productivity. However, if sectors fail to coordinate on adopting – when there is no hype – there is no fee collected for the innovation. Since hype is persistent, an innovator that observes hype today expects to license the frontier after a success under hype as well, on the greatest equilibrium, where the fee is higher. Hype therefore raises the innovator's value.

Research depends on the difference between the value after a success and after a failure, so hype raises research when it raises the first by more than the second. Hype raises the value of a success but also the value of a failure, because the innovator can license  $q_s^{\delta_s}$ , a narrower but still profitable gap. However, if  $\delta_s$  is sufficiently small, upon failure, the gap drops close to one and the fee is close to zero. Hype then raises the value after a success by much more than the value

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<sup>8</sup>Uniqueness of the research equilibrium can also be established under an assumption on primitives. When the discount factor, the fee, and the response of the success probability to the success premium are small enough, the map from the other innovators' success probabilities to an innovator's own is a contraction. The research equilibrium is then unique. Assumption B.1 replaces this condition with the selection of the greatest equilibrium.

after a failure, and the innovator exploits it by increasing research, and hence the probability of a success. The following theorem formalizes this.

**Theorem 3.** *Suppose standalone returns are negative in every sector and hype is persistent,  $\pi_{00} + \pi_{11} > 1$ . Fix the other sectors' research. Then, for every innovator  $s$ :*

- (i) *Hype weakly raises the innovator's value:  $v_s(\mathbf{q}, 1) \geq v_s(\mathbf{q}, 0)$  for every  $\mathbf{q} \in \mathcal{Q}$ .*
- (ii) *At complete diffusion,  $\delta_s = 0$ , hype weakly raises research:  $r_s^*(\mathbf{q}, 1) \geq r_s^*(\mathbf{q}, 0)$  for every  $\mathbf{q} \in \mathcal{Q}$ . If  $\inf_{\mathbf{q} \in \mathcal{Q}} [r_s^*(\mathbf{q}, 1) - r_s^*(\mathbf{q}, 0)] > 0$  at  $\delta_s = 0$ , the inequality holds, with the same research of the other sectors, for all sufficiently small  $\delta_s > 0$ .*

The theorem holds the other sectors' research fixed and thus compares the research of innovator  $s$  alone. Under hype the other innovators research more as well, their successes widen their gaps and raise the support sector  $s$  receives. Section 4.3 shows that this response reinforces the effect.

### 4.3 Endogenous closure and reopening

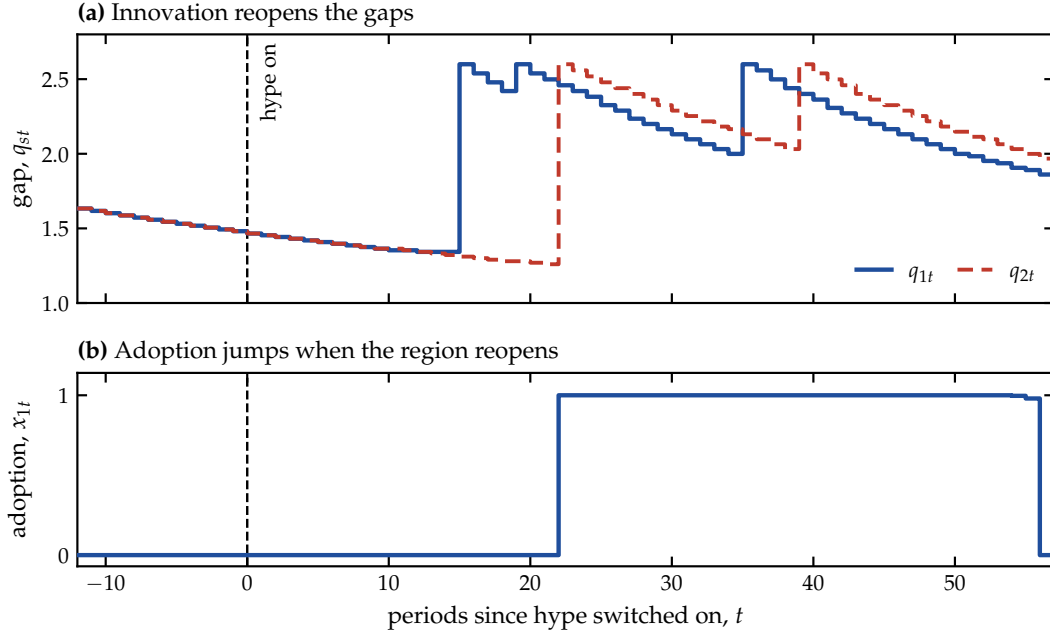
Theorem 3 shows that hype raises research, and research in turn determines whether the gap vector lies in the region of multiple equilibria, where hype changes adoption (Section 3.3). A success weakly raises every entry of row  $s$  of the effective network relative to a failure, since  $\gamma_s \geq q_s^{\delta_s}$ , and with it the multiplier. To illustrate how the two effects combine, I use the two-sector economy of Section 3.3, first under a permanent absence of hype and then under permanent hype.

Start from panel (b) of Figure 3, where the gaps are wide, both best responses are steep, and the least and greatest equilibria are no adoption and full adoption. Suppose that hype is absent and expected to stay absent. Producers coordinate on no adoption and the fee is zero in both sectors. With hype expected to stay absent, each innovator will license on the least equilibrium in every future period, where the fee is zero whatever the gap, so a success is worth the same as a failure and research is zero in both sectors. Zero research is optimal whatever the other innovator does, and the research equilibrium is unique.

The frontier is then unchanged while basic productivity diffuses toward it, closing a fixed fraction of the log gap each period. As the gaps narrow, adopting the frontier puts less productivity to use, so the same support from the other sector has a smaller effect on the return to adoption. The slope of the best response falls, because the gain from adoption is measured in units of that productivity while the adjustment cost is measured in units of basic productivity (Section 2). The effective spillovers fall and both best responses become flatter, as in panel (a).

A narrower gap lowers each best response at every support, so the greatest equilibrium is nonincreasing along this path. In the two-sector economy, full adoption remains the greatest equilibrium as long as each sector's best response to full adoption by the other is full adoption (Lemma 2). Once the gaps fall below that level, best-response iteration from full adoption converges to no adoption and the greatest equilibrium drops to no adoption at once. In general,

**Figure 4: TWO SECTORS: ENDOGENOUS REOPENING AFTER HYPE**



*Notes:* One simulated path of the economy of Figures 2 and 3. Before the dashed line, the economy has been without hype long enough for research to stop and diffusion to narrow the gaps (Proposition 1). Hype switches on at the dashed line and remains on. Panel (a) shows the two productivity gaps, solid for sector 1 and dashed for sector 2. Panel (b) shows adoption on the selected equilibrium. Adoption responds once successes have moved the gap vector into the region of multiple equilibria. For the parametrization used, see Appendix C.4.

the gap vector leaves the region of multiple equilibria at the latest when the multiplier falls below one (Theorem 1). Figure 4 plots one path of this economy. Before the dashed line, hype has been absent long enough for research to be zero and for both gaps in panel (a) to fall below the level at which the greatest equilibrium has positive adoption, and adoption in panel (b) is zero. The following proposition formalizes this path.

**Proposition 1 (Closure).** *Suppose standalone returns are negative in every sector,  $\mathbf{q}_0$  lies in the region of multiple equilibria,  $z_0 = 0$ , and  $\pi_{00} = 1$ . Then the equilibrium path is unique,  $\bar{\mathbf{x}}(\mathbf{q}_t)$  and  $\text{NAM}(\mathbf{q}_t)$  are nonincreasing in  $t$ , and there is a finite  $t^*$  such that  $\text{NAM}(\mathbf{q}_t) < 1$  and the adoption equilibrium is unique for every  $t \geq t^*$ .*

Along this path research is zero and each gap narrows deterministically,  $q_{s,t} = q_{s,0}^{\delta_s^t} \rightarrow 1$ . The date  $t^*$  is an upper bound on the date at which the gap vector leaves the region, and it depends on the initial gaps, the diffusion parameters, the spillovers, and the adjustment costs alone. Wider initial gaps, slower diffusion, and stronger spillovers raise it.

Consider now Figure 4 after the dashed line, where hype returns and is expected to stay. The return is unanticipated, since before the dashed line innovators expected hype to stay absent. Adoption in panel (b) stays at zero. Hype coordinates producers on the greatest equilibrium, and at these gaps the greatest equilibrium is no adoption. Full support from the other sector would

lead each sector to adopt part of its frontier, because the support covers the standalone loss, but partial adoption returns less support than it receives and each round of feedback is weaker than the previous one. The NAM is below one and no adoption is the unique equilibrium (Theorem 1).

Research, in contrast, rises as soon as hype returns. The return of hype leaves today's fee at zero but changes what a success is worth. A success widens the innovator's gap and makes its best response steeper. Once successes have widened the gaps enough, the greatest equilibrium has positive adoption, the fee is positive, and with hype expected to persist the innovator expects to earn it. The success premium is therefore positive again, and both innovators research from the date hype returns. In panel (a), sector 1 succeeds first. Its gap jumps to  $\gamma_1$  while sector 2's gap keeps narrowing, and adoption stays at zero, because at sector 2's narrow gap its best response is flat and best-response iteration from full adoption converges to no adoption. Adoption jumps to full adoption in panel (b) when sector 2 succeeds as well.

In general, each success raises one row of the effective network, as widening the gaps in Figure 3 raises both rows, and successes in several sectors may be needed before the gap vector enters the region of multiple equilibria and any innovator earns a fee. An innovator researches even when its own success leaves the gap vector outside the region, because a success today keeps its gap wide, net of diffusion, during the periods in which the other sectors succeed.

This delayed response is the *hysteresis*. Hype coordinates producers on an equilibrium that exists at today's gaps, while it raises research through the fee expected at tomorrow's gaps. After a long enough spell without hype, diffusion has narrowed the gaps, and successes must widen them before adoption starts. The delay between the return of hype and the return of adoption is the time successes need to move the gap vector into the region of multiple equilibria. The longer the spell without hype, the narrower the gaps when hype returns and the more successes are needed, up to one per sector.

Between successes, diffusion narrows the gaps, so the multiplier rises in expectation when successes are frequent enough. At the end of the path in Figure 4, both gaps have narrowed again between successes and adoption returns to zero until the next success. Since the least equilibrium is no adoption at every gap vector, the region of multiple equilibria is the set of gap vectors at which the greatest equilibrium has positive adoption. The following proposition formalizes the reopening.

**Proposition 2.** *Suppose standalone returns are negative in every sector and multiple equilibria are possible for some  $\mathbf{q}$ .*

- (i) *Let  $\tau_s(q)$  be the smallest success probability at which, from  $q_{st} = q$ , the expected log of  $q_{s,t+1} - 1$  is at least  $\log(q - 1)$ . If  $h_s(r_{st}) \geq \tau_s(q_{st})$  for every  $s$ , then  $\mathbb{E}[\text{NAM}(\mathbf{q}_{t+1}) \mid \mathbf{q}_t, z_t] \geq \text{NAM}(\mathbf{q}_t)$ .*
- (ii) *Suppose hype is permanent and  $h_s(r_{st})$  is bounded away from zero for every  $s$  whenever  $\mathbf{q}_t$  lies outside the region. Then  $\mathbf{q}_t$  enters the region in finite time with probability one.*

The threshold  $\tau_s$  compares a success with diffusion. In each period diffusion narrows the productivity available to be adopted,  $q - 1$ , while a success raises it to  $\gamma_s - 1$ , and  $\tau_s$  is the success probability at which the two effects offset each other in expectation, so that above it the expected

$\log$  of  $q - 1$  rises. The multiplier is nondecreasing in each gap and its  $\log$  is a convex function of the  $\log$ s of the  $q_s - 1$  (Appendix A.5), so the expected multiplier rises as well. In the two-sector economy the  $\log$  of the multiplier is the average of the two  $\log$ s of  $q_s - 1$  plus a constant, and the multiplier rises in expectation whenever the expected  $\log$  of each  $q_s - 1$  does. The threshold is close to zero at a narrow gap, where a failure narrows the gap by little and a success raises  $q - 1$  by a large multiple. It equals one at  $q_s = \gamma_s$ , where a success keeps the gap at  $\gamma_s$  and every failure narrows it, and at complete diffusion, where a success alone keeps the gap above one.

Part (ii) gives conditions under which the gap vector enters the region in finite time. Under permanent hype with research bounded away from zero in every sector, all sectors succeed in the same period with positive probability, which moves every gap to  $\gamma_s$  and the gap vector into the region.

Section 5 maps these research and productivity dynamics into sectoral and aggregate growth.

## 5 Growth

How much of long-run growth does coordination decide? Coordination affects both how much of the frontier is adopted and how often the frontier advances. This section maps these two channels into long-run growth. Adoption is bounded, so its direct effect on productivity is a level effect that washes out of stationary growth; the permanent channel operates through innovation (Theorem 4). I then decompose the growth difference between the regimes with and without hype into innovation and adoption components (Theorem 5). Input–output shares aggregate sectoral productivity growth into consumption growth with fixed Cobb–Douglas weights.

### 5.1 From productivity to growth

Write  $a_s^*(\mathbf{q}_t, z_t)$  for the productivity ratio  $A_{st}/A_{st}^B$  in (7), evaluated at the adoption profile selected by hype. Effective productivity then decomposes as

$$\log A_{st} = \log A_{st}^B + \log a_s^*(\mathbf{q}_t, z_t).$$

The first component accumulates as basic productivity advances; the second captures the productivity obtained from adoption and is bounded.

By (14), basic productivity grows by  $\log q_{st}$  after a success and by  $(1 - \delta_s) \log q_{st}$  after a failure. Its conditional expected growth is therefore

$$[1 - \delta_s + \delta_s h_s(r_s^*(\mathbf{q}, z))] \log q_s.$$

Fixed Cobb–Douglas shares map sectoral productivity into consumption. With the cost-based

Domar weights  $\tilde{\lambda}$  derived in Appendix B.1, consumption growth satisfies the exact identity

$$\Delta \log C_{t+1} = (1 - \alpha) \sum_{s=1}^S \tilde{\lambda}_s \left\{ \Delta \log A_{s,t+1}^B + \log a_s^*(\mathbf{q}_{t+1}, z_{t+1}) - \log a_s^*(\mathbf{q}_t, z_t) \right\}. \quad (21)$$

The first term is the advance of basic productivity, which accumulates; the remaining terms are the change in the bounded adoption component. The Domar weights incorporate the direct and downstream effects of sectoral productivity through the input–output network. The corresponding sectoral output and price mappings are collected in Appendix C.2.

Conditional expected consumption growth follows from (21) by replacing  $\Delta \log A_{s,t+1}^B$  with its conditional expectation and averaging the change in  $\log a_s^*$ .

## 5.2 Long-run growth

Equation (21) separates the transitory and permanent effects of coordination. Selecting the greatest equilibrium raises productivity by putting more of the frontier to use, but this direct effect is bounded and therefore has zero stationary mean growth. Research instead changes how often frontiers advance and basic productivity grows. Coordination can therefore affect the long-run growth rate through its effect on innovation.<sup>9</sup>

**Theorem 4.** *Let  $\mathbb{P}^*$  be an invariant law of the selected Markov policy. Stationary mean consumption growth is*

$$g_C^* = (1 - \alpha) \int_{Q \times \{0,1\}} \sum_{s=1}^S \tilde{\lambda}_s [1 - \delta_s + \delta_s h_s(r_s^*(\mathbf{q}, z))] \log q_s d\mathbb{P}^*(\mathbf{q}, z). \quad (22)$$

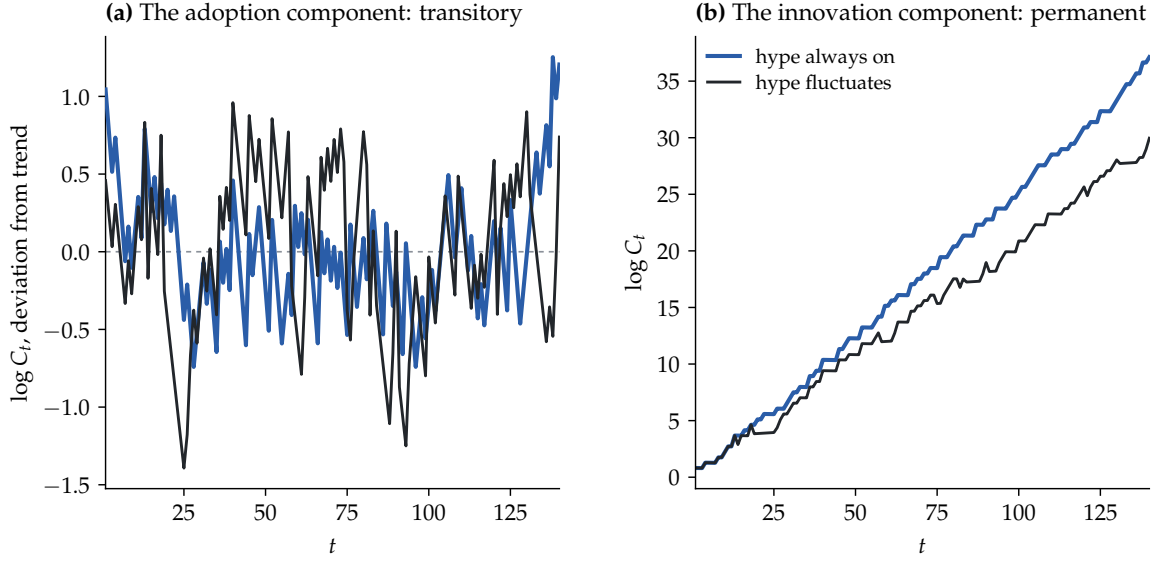
*If  $\mathbb{P}^*$  is ergodic and the initial state follows  $\mathbb{P}^*$ ,  $g_C^*$  is also the almost-sure time average.*

Theorem 4 shows how coordination reaches long-run growth through research. The integrand in (22) is the expected advance of sector  $s$ 's basic productivity. After a failure, diffusion closes the fraction  $1 - \delta_s$  of the log gap; after a success, the entire gap is absorbed into basic productivity and a new gap opens. A success therefore adds the remaining fraction  $\delta_s \log q_s$ , and occurs with probability  $h_s(r_s^*)$ . Adoption does not appear directly because it changes a bounded component of productivity. It enters through  $r_s^*(\mathbf{q}, z)$ : the adoption equilibrium determines license income and hence the probability of innovation.

Figure 5 shows both effects of coordination in the two-sector example, with one hype path and one set of research outcomes for the two economies. Panel (a) removes each economy's own trend. When hype fluctuates, sectors switch between the least and greatest adoption equilibria, and output swings around its trend with a standard deviation of 0.52, a third larger than the 0.39 under permanent hype. These swings average zero, because switching equilibria moves only the bounded adoption component. Their cost appears in panel (b). Every period without hype is a

<sup>9</sup>An invariant law for  $(\mathbf{q}, z)$  exists and is unique whenever research is bounded away from zero and either  $0 < \pi_{00}, \pi_{11} < 1$  or the regime is permanent; the law of the state converges to it from any initial condition (Lemma B.2 in Appendix B.6).

**Figure 5:** TRANSITORY ADOPTION AND PERMANENT INNOVATION



*Notes:* Two-sector example with Markov perfect research policies. Panel (a) shows the deviation of  $\log C_t$  from each economy's own linear trend; both series have mean zero, with standard deviation 0.52 when hype fluctuates and 0.39 under permanent hype. Panel (b) shows the levels, with trend growth 0.208 under fluctuating hype and 0.260 under permanent hype. The two economies use one hype path and one set of research outcomes. For the parametrization used, see Appendix C.4.

period of low license income and fewer innovation successes, so the switches that move output around its trend also lower the trend itself: growth is 0.208 under fluctuating hype against 0.260 under permanent hype, a fifth lower. The swings in adoption are transitory; the innovation they stall is lost for good, and the gap in levels widens every period.

The permanent absence of hype is the limit of this mechanism. With negative standalone returns (Corollary C.1(ii)), sectors coordinate on no adoption in every period, research is zero, diffusion drives  $\mathbf{q}$  to 1, and (22) gives  $g_C^* = 0$ : an economy that permanently coordinates on no adoption eventually stops growing.

Sectoral growth rates are generally heterogeneous, and a common growth rate requires mean productivity growth to align with the input–output structure. The corresponding mappings for sectoral quantities, prices, and wages are collected in Appendix C.2. One implication matters below: relative wages are pinned by shares, so coordination affects wages through aggregate consumption.

### 5.3 Coordination and growth

Coordination affects growth through two margins. First, it changes research, and therefore how often frontiers advance. Second, it changes adoption, hence how much of the available frontier is used. Theorem 5 separates these margins.

For  $b \in \{0, 1\}$ , let  $r_s^b(\mathbf{q})$  denote the research policy under the permanent regime  $z_t \equiv b$ , let

$\mathbb{P}^b$  denote an invariant law of that regime when one exists, and let  $\mathbb{E}^b[\cdot \mid \mathbf{q}]$  denote conditional expectations under its research policy and selection rule.

**Theorem 5.** *At every common gap vector  $\mathbf{q}$ ,*

$$\begin{aligned} \mathbb{E}^1[\Delta \log C_{t+1} \mid \mathbf{q}] - \mathbb{E}^0[\Delta \log C_{t+1} \mid \mathbf{q}] &= (1 - \alpha) \left\{ \underbrace{\sum_s \tilde{\lambda}_s \delta_s [h_s(r_s^1(\mathbf{q})) - h_s(r_s^0(\mathbf{q}))]}_{\text{innovation}} \log q_s \right. \\ &\quad \left. + \underbrace{\sum_s \tilde{\lambda}_s \left\{ \mathbb{E}^1[\log a_s^*(\mathbf{q}', 1) \mid \mathbf{q}] - \log a_s^*(\mathbf{q}, 1) - \mathbb{E}^0[\log a_s^*(\mathbf{q}', 0) \mid \mathbf{q}] + \log a_s^*(\mathbf{q}, 0) \right\}}_{\text{adoption}} \right\}. \end{aligned} \quad (23)$$

*For invariant laws  $\mathbb{P}^1$  and  $\mathbb{P}^0$  of the permanent regimes, the long-run growth difference is*

$$g_C^1 - g_C^0 = (1 - \alpha) \sum_s \tilde{\lambda}_s \left\{ \underbrace{\int \delta_s [h_s(r_s^1) - h_s(r_s^0)] \log q_s d\mathbb{P}^0}_{\text{research policy}} + \underbrace{\int [1 - \delta_s + \delta_s h_s(r_s^1)] \log q_s d(\mathbb{P}^1 - \mathbb{P}^0)}_{\text{gap distribution}} \right\}. \quad (24)$$

The two expressions characterize the same growth difference at different horizons. At a common gap vector, (23) separates the one-period effect into innovation and adoption. The innovation term is the difference in success probabilities times the part of the log gap that diffusion would otherwise leave open. The adoption term is the difference in the change of the transitory component under the two different sunspot realizations.

Equation (24) separates the long-run growth difference into a research-policy effect and a distributional effect. The first evaluates the difference in success probabilities at the no-hype distribution  $\mathbb{P}^0$ . With negative standalone returns in every sector,  $\mathbb{P}^0$  is the point mass at  $\mathbf{q} = \mathbf{1}$  (Lemma B.2), the research-policy effect is zero, and the whole growth difference is distributional. The second captures the change from  $\mathbb{P}^0$  to  $\mathbb{P}^1$  in the states at which research and diffusion operate. Thus coordination can affect long-run growth both by changing research at a given gap vector and by changing the gap vectors the economy visits.

Next, Section 6 and Section 7 estimate the adoption network and quantify the total growth effect from coordination.

## 6 Measuring Adoption Interactions

This section estimates how strongly a sector's adoption responds to adoption by the sectors that support it. These estimates are the targets that discipline the quantitative model of Section 7.

Three problems arise. The first is dimensionality: with  $S$  sectors there are  $S(S - 1)$  unknown links. The second is reflection (Manski, 1993): the adoption of linked sectors comoves under common shocks as well as under adoption complementarities. The third is that adoption lies between zero and one, with many observations at each bound, where a linear regression understates the response. I measure the links from predetermined technology portfolios, in the tradition of the

peer-effects literature (Bramoullé et al., 2020), so that only the scale of each sector’s response remains to be estimated. I identify that scale with a granular instrument (Gabaix and Koijen, 2024): idiosyncratic technology hiring by large firms moves the adoption of their sector independently of shocks to the sectors linked to it.<sup>10</sup> And I estimate each sector’s response with a two-limit Tobit, which has the form of the model’s best response.

## 6.1 Data

Testing for coordination failure requires the adoption of many technologies by many sectors, year after year, which classic diffusion studies of one technology at a time (Griliches, 1957; Mansfield, 1961) and the few waves of the World Bank’s FAT survey (Cirera et al., 2022) leave uncovered. I measure adoption from the technology content of online job postings (Kalyani et al., 2025). The technologies are two-word phrases identified from patent text and earnings calls and hand-audited into a final list of 387. A posting that mentions a technology reveals that a firm is hiring people to work with it, and I assign each posting to the firm’s four-digit NAICS sector. The panel covers 305 sectors in 2007 and 2010–2019; the years 2007, 2010, and 2011 define each sector’s baseline technology portfolio, and 2012–2019 is the outcome window.

I treat each of the  $K = 387$  technologies as a separate play of the adoption game, with links  $\theta_{ss'}$  and standalone returns  $\kappa_s$  common across technologies and productivity gaps specific to each. Within each play, the counterpart of adoption  $x_s$  is  $x_{skt} \in [0, 1]$ : the share of sector  $s$ ’s postings in year  $t$  that mention technology  $k$ , divided by a technology-specific reference share and capped at one.<sup>11</sup> This normalization puts technologies with different baseline frequencies on a common intensity scale.

## 6.2 Mapping from the model to the data

Consider an economy with three sectors and three technologies,  $a$ ,  $b$ , and  $c$ . For each technology  $k$ , the best response (16) of sector 1 is

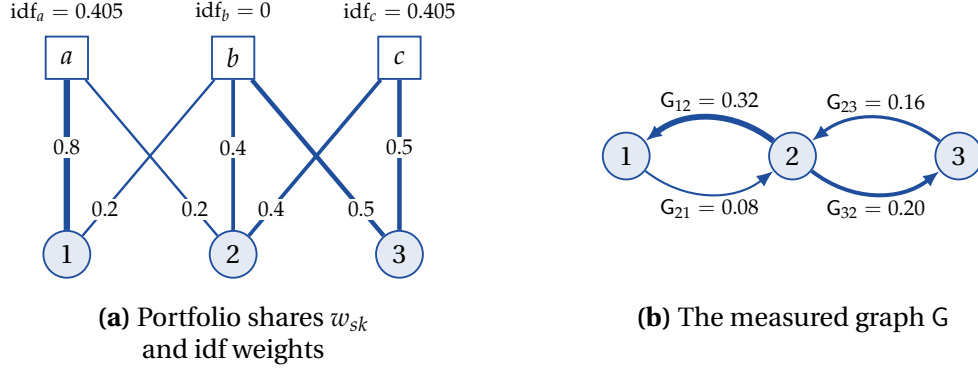
$$x_{1k} = \max \left\{ \min \left\{ \frac{q_{1k} - 1}{\chi_1} [\kappa_1 + \theta_{12}x_{2k} + \theta_{13}x_{3k}], 1 \right\}, 0 \right\}, \quad (25)$$

where  $q_{1k}$  is sector 1’s gap for technology  $k$ . Adoption  $x_{1k}$ ,  $x_{2k}$ , and  $x_{3k}$  is observed in the postings. I measure the links  $\theta_{12}$  and  $\theta_{13}$  from the overlap of the portfolios, because sectors that use the same technologies benefit from each other’s adoption, in the same way that the overlap of patent portfolios measures technological proximity (Jaffe, 1986; Bloom et al., 2013). Sectors 2 and 3 support sector 1 when they use technologies important to sector 1. Each technology is weighted

<sup>10</sup>Similar designs identify the spread of optimism across firms (Flynn and Sastry, 2025) and the spillover of large borrowers’ shocks to other borrowers of the same bank (Galaasen et al., 2025).

<sup>11</sup>The reference share is the 95th percentile of the technology’s positive shares across sectors and years, or the maximum for thinly observed technologies.

**Figure 6: HOW G IS BUILT**



*Notes:* Three sectors (circles) and three technologies (squares). Panel (a) shows each sector's baseline technology portfolio, with lines labeled by  $w_{sk}$ , and each technology's inverse-frequency weight. Panel (b) shows the resulting directed links  $G_{ss'}$ .

by the sector's reliance on it, its share of the sector's baseline technology portfolio,

$$w_{sk} = \frac{x_{sk,07-11}}{x_{sa,07-11} + x_{sb,07-11} + x_{sc,07-11}}, \quad s = 1, 2, 3, \quad k = a, b, c,$$

where  $x_{sk,07-11}$  is sector  $s$ 's adoption of  $k$  averaged over 2007, 2010, and 2011, and by its rarity across sectors, the inverse-frequency weight  $\text{idf}_k = \log(S_0/n_k)$ , the log of the number of sectors  $S_0$  over the number  $n_k$  whose baseline portfolio includes  $k$ . Rare technologies receive greater weight because their shared use provides stronger evidence of sectoral support, much as rare words are more informative about a document than common ones (Kelly et al., 2021). The links into sector 1 add these weights over the technologies in the source sector's baseline portfolio,

$$\begin{aligned} G_{12} &= w_{1a} \text{idf}_a e_{2a} + w_{1b} \text{idf}_b e_{2b} + w_{1c} \text{idf}_c e_{2c}, \\ G_{13} &= w_{1a} \text{idf}_a e_{3a} + w_{1b} \text{idf}_b e_{3b} + w_{1c} \text{idf}_c e_{3c}, \end{aligned}$$

with  $e_{2k} = \mathbf{1}\{x_{2k,07-11} > 0\}$  recording that  $k$  is in sector 2's baseline portfolio, and likewise  $e_{3k}$ . Figure 6 illustrates how G is constructed.<sup>12</sup>

Plugging the graph into the best responses, for every technology  $k$ ,

$$\begin{aligned} x_{1k} &= \max\{\min\{a_{1k} + \beta_1 (G_{12} x_{2k} + G_{13} x_{3k}), 1\}, 0\}, \\ x_{2k} &= \max\{\min\{a_{2k} + \beta_2 (G_{21} x_{1k} + G_{23} x_{3k}), 1\}, 0\}, \\ x_{3k} &= \max\{\min\{a_{3k} + \beta_3 (G_{31} x_{1k} + G_{32} x_{2k}), 1\}, 0\}, \end{aligned}$$

where the *loading*  $\beta_s$ , the strength of sector  $s$ 's response to support, is the one number per sector left to estimate and averages the scale  $(q_{sk} - 1)/\chi_s$  over technologies, and  $a_{sk}$  collects  $\beta_s \kappa_s$  and the technology-specific part of the gap. The links are common to all technologies, including  $b$ ,

<sup>12</sup>See Appendix D.1 for details on the construction of the figure

since they describe how connected the technology use of two sectors is, which the whole portfolio reveals. The matrix  $\text{diag}(\beta)G$  is the *estimated effective network*, the counterpart of the effective network  $\hat{\Theta}(\mathbf{q})$  at the gaps observed in the data, and the multiplier is its spectral radius. In this graph support travels along two cycles, between sectors 1 and 2 and between sectors 2 and 3, and the spectral radius is the square root of the sum, over the two cycles, of the product of the loadings and links around the cycle,  $\sqrt{\beta_1\beta_2G_{12}G_{21} + \beta_2\beta_3G_{23}G_{32}}$ .

In general, for sector  $s$  and technology  $k$ ,

$$x_{sk} = \max\left\{\min\left\{a_{sk} + \beta_s \sum_{s' \neq s} G_{ss'} x_{s'k}, 1\right\}, 0\right\}, \quad (26)$$

the best response (16) with the measured graph in place of the network, a linear-in-means model of peer effects with a censored outcome (Bramoullé et al., 2009, 2020), in which a sector's adoption responds linearly to a weighted average of the other sectors' adoption and the bounds censor it. The graph fixes the direction and the relative strength of the links into each sector, and  $\beta_s G_{ss'}$  is the counterpart of the effective spillover  $(q_{sk} - 1)\theta_{ss'}/\chi_s$ . The intercept  $a_{sk}$  is the counterpart of the effective standalone return  $(q_{sk} - 1)\kappa_s/\chi_s$ , with the same scale. I call sector  $s$  the *receiver* and the sectors that support it its *sources*, and the support receiver  $s$  receives for technology  $k$  is the graph-weighted adoption of its sources,  $\sum_{s'} G_{ss'} x_{s'k}$ . Appendix D.4 re-estimates the NAM on alternative graphs built from equipment investment, common customers, market returns, occupations, patents, and sales. Each of them is denser and covers fewer sectors, so the quantitative results rest on the most conservative network.

### 6.3 Identifying adoption responses

I estimate the coefficient  $\beta_s$  through the annual panel, that provides up to eight years of observations on 387 technologies per sector. Sector 1's adoption of technology  $k$  in year  $t$  is

$$x_{1kt} = \min\left\{\max\left\{a_1 + \beta_1(G_{12}x_{2kt} + G_{13}x_{3kt}) + \gamma_1^0 x_{1k,07-11} + \tau_{1t} + e_{1kt}, 0\right\}, 1\right\}, \quad (27)$$

where  $x_{1k,07-11}$  is the predetermined adoption of sector 1 for  $k$ ,  $\tau_{1t}$  is the year fixed effect, absorbs shocks and  $e_{1kt}$  is the error. A shock to technology  $k$  raises adoption in sector 1 and in sectors 2 and 3 at once. I isolate the variation in support that originates with sectors 2 and 3 from the hiring of their large firms (Gabaix and Koijen, 2024). For each firm  $i$ , technology  $k$ , and year  $t$ , I regress the firm's hiring for technology  $k$  on firm-year effects, technology-year effects, and firm-technology effects, additionally adjusting for firm size within each technology-year entry. Thus, the residual  $\hat{\varepsilon}_{ikt}$  is the firm's abnormal hiring for technology  $k$  in year  $t$ . I then sum up the residuals within each sector, weighting each firm by its predetermined share of its sector's size over 2007-2009,  $\bar{w}_i$ . Sector 2's shock is  $\eta_{2kt} = \sum_{i \in 2} \bar{w}_i \hat{\varepsilon}_{ikt}$ , and sector 3's is  $\eta_{3kt}$ . Sector 1's instrument weights the two shocks by its links,  $z_{1kt} = G_{12}\eta_{2kt} + G_{13}\eta_{3kt}$ . The first stage regresses sector 1's support on its

instrument,

$$G_{12}x_{2kt} + G_{13}x_{3kt} = \psi_1 z_{1kt} + \psi_1^0 x_{1k,07-11} + \tau_{1t}^1 + v_{1kt}.$$

Adoption is censored at zero and one, so the Tobit is nonlinear in support. In a linear regression, the fitted support from the first stage could replace support. In the Tobit, the censoring applies to actual support, so replacing it by its fitted value gives inconsistent estimates. The control function (Smith and Blundell, 1986) keeps actual support in the Tobit and adds the first-stage residual  $\hat{v}_{1kt}$  as a regressor,

$$x_{1kt} = \min \left\{ \max \left\{ a_1 + \beta_1 (G_{12}x_{2kt} + G_{13}x_{3kt}) + \gamma_1^0 x_{1k,07-11} + \tau_{1t} + \rho_1 \hat{v}_{1kt} + \epsilon_{1kt}, 0 \right\}, 1 \right\}.$$

In general, for receiver  $s$ , the adoption equation is the two-limit Tobit<sup>13</sup>

$$x_{skt} = \min \left\{ \max \left\{ a_s + \beta_s \underbrace{\sum_{s'} G_{ss'} x_{s'kt}}_{\text{support}} + \underbrace{\gamma_s^0 x_{sk,07-11}}_{\text{baseline}} + \underbrace{\tau_{st}}_{\text{year effects}} + e_{skt}, 0 \right\}, 1 \right\}, \quad (28)$$

the source shocks and the instrument are

$$\eta_{s'kt} = \sum_{i \in s'} \bar{w}_i \hat{\epsilon}_{ikt}, \quad z_{skt} = \sum_{s'} G_{ss'} \eta_{s'kt}, \quad (29)$$

the first stage is

$$\underbrace{\sum_{s'} G_{ss'} x_{s'kt}}_{\text{support}} = \psi_s \underbrace{z_{skt}}_{\text{instrument}} + \psi_s^0 x_{sk,07-11} + \tau_{st}^1 + v_{skt}, \quad (30)$$

and the second stage is

$$x_{skt} = \min \left\{ \max \left\{ a_s + \beta_s \underbrace{\sum_{s'} G_{ss'} x_{s'kt}}_{\text{support}} + \gamma_s^0 x_{sk,07-11} + \tau_{st} + \underbrace{\rho_s \hat{v}_{skt}}_{\text{control function}} + \epsilon_{skt}, 0 \right\}, 1 \right\}, \quad (31)$$

which I estimate by maximum likelihood. The identifying restriction is that the idiosyncratic hiring of source firms affects a receiver's adoption only through adoption in the source sector.

**Assumption 1.** Let  $\mathcal{C}_{sk}$  collect baseline adoption, the year effects, the graph, and the firm weights. For every receiver  $s$ , technology  $k$ , year  $t \in \{2012, \dots, 2019\}$ , and source firm  $i \notin s$  that contributes to  $z_{skt}$ ,

$$\mathbb{E}[\hat{\epsilon}_{ikt} e_{skt} \mid \mathcal{C}_{sk}] = 0.$$

**Example 1.** Boeing belongs to aerospace product and parts manufacturing. Over 2007-2009 it accounted for 13 percent of the listed-firm size of its sector, which fixes its weight  $\bar{w}_i$ . In 2016 it posted 131 rapid-prototyping positions, against 9 the year before and 31 the year after. The

<sup>13</sup>For details, see Tobin (1958) and Wooldridge (2010, Chapter 17).

**Table 1:** Adoption responses and the multiplier

|                                                    | Control-function Tobit |
|----------------------------------------------------|------------------------|
| Median loading $\hat{\beta}_s$                     | 0.090                  |
| Median intercept $\hat{a}_s$                       | −0.474                 |
| Median control-function coefficient $\hat{\rho}_s$ | −0.009                 |
| Adoption response to a 1% increase in support      | 0.72%                  |
| Median first-stage $F$                             | 68                     |
| Receivers                                          | 277                    |
| $\widehat{NAM}$                                    | 1.179                  |
|                                                    | [1.076, 1.255]         |

*Notes:* Receiver-specific estimates of (30)–(31) on the annual panel, 2012–2019, for the 277 supported receivers of the 278-sector quantitative model. Medians are across receivers. The adoption response is the median average partial effect of a one percent increase in mean support, as a percentage of mean adoption.

firm-year effect absorbs Boeing’s overall hiring in 2016. The technology-year effect absorbs the demand for rapid prototyping across all firms in 2016. The firm-technology effect absorbs its usual hiring for rapid prototyping. The jump that remains is Boeing’s unusual hiring, which, after weighted by 13 percent, enters the shock of aerospace manufacturing. Through the links it raises the support of every sector that aerospace manufacturing supports. Assumption 1 requires that the jump provides support to other sectors only through the fact that it increases granuarly its sector’s adoption.

Appendix D.2 gives the estimation details.

## 6.4 Results

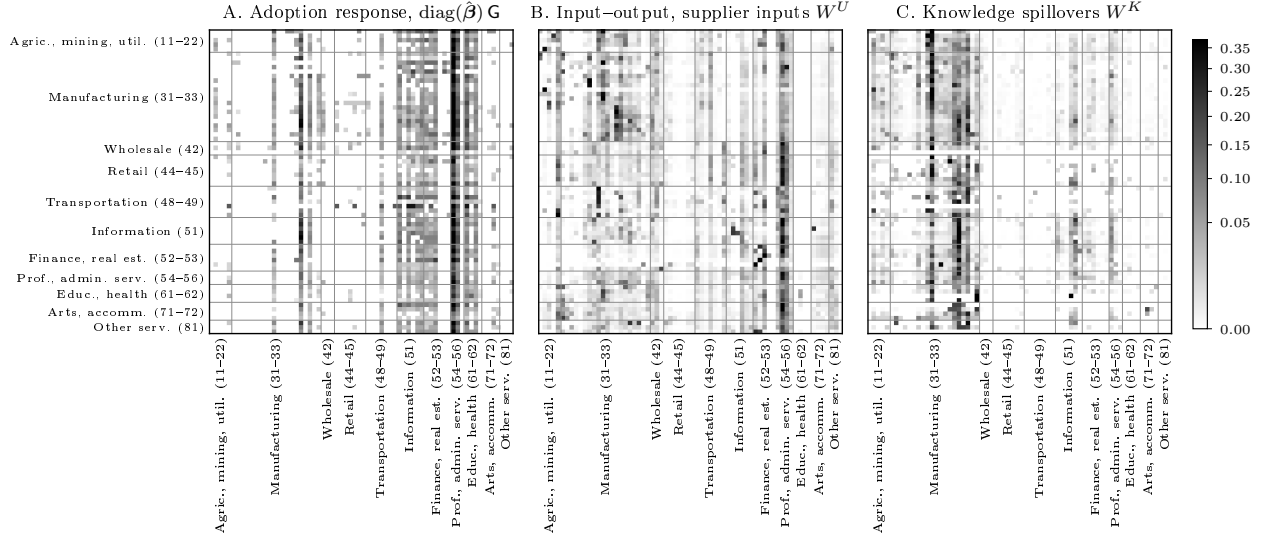
Adoption responds strongly to support. A one percent increase in support raises the adoption of the median receiver by 0.72 percent, and by 0.55 to 0.91 percent across the interquartile range of receivers.<sup>14</sup> A receiver’s adoption therefore moves almost in proportion with the adoption of its sources.

The response begins only once support is large. With zero baseline adoption, the median receiver adopts a technology once its support exceeds 5.3, three times its mean support. A single source at full adoption supplies 0.6 units of support through an average link, and 72 percent of receiver–technology–year observations show zero adoption. A receiver therefore adopts when many of its sources adopt at once.

Complementarities are strong enough to satisfy the necessary condition for multiple equilibria. The empirical Network Adoption Multiplier  $\widehat{NAM} = 1.179$ . A unit of adoption spread across sectors in proportion to the leading eigenvector of the estimated effective network returns as 1.18

<sup>14</sup>The effect is the average partial effect of the Tobit, the loading times the average probability of adoption between the bounds, evaluated at a one percent increase in the receiver’s mean support and expressed as a percentage of its mean adoption.

**Figure 7: A COMPARISON ACROSS THE THREE NETWORKS**



*Notes:* Panel A is the estimated effective network  $\text{diag}(\hat{\beta})G$ , panel B the supplier-input network, and panel C the knowledge network, on the 203 sectors common to the three. Rows are receivers and columns sources, in NAICS order; lines separate two-digit sectors, labeled by code, with small neighboring sectors sharing a label. Each cell averages, over the four-digit receivers in its row group, the sum of links to the four-digit sources in its column group, for 68 three-digit groups.

units after one round of mutual support, so successive rounds grow wherever adoption lies inside the bounds.

**The estimated effective network.** Figure 7, panel A, shows the estimated effective network aggregated at NAICS3 level. A few source columns carry most of the links: professional, scientific, and technical services (28 percent of all links), information (18), finance (10), computer and electronic products (9), administrative services (8), education (7), and health (6). Within these groups, the largest sources are the sectors whose business is to develop, implement, and teach technologies: engineering services, scientific research and development, management consulting, computer systems design, software publishers, data processing and hosting, employment services, and colleges and universities. These sources support most receivers. Across the 277 receivers of the quantitative model, engineering services is among the five largest sources of 196, and research and development services, employment services, and computer systems design are each among the five largest sources of about 150. The adoption of a technology anywhere in the economy thus depends on its adoption by the sectors that hold and transmit the know-how to use it.

The literature measures links between sectors with networks that record other flows: input-output proximity predicts which inputs a sector adopts (Carvalho and Voigtländer, 2014), technology transitions propagate along supply chains (Aghion et al., 2025), and knowledge spillovers follow patent citations (Liu and Ma, 2024). The input-output network weights a source by its share in the receiver's costs and the knowledge network by the receiver's citations to its patents, and the sectors that carry adoption in the estimated effective network weigh little on either:

**Table 2:** Adoption response conditional on input–output and knowledge networks

|                                | (1)                   | (2)                  | (3)                   | (4)                   | (5)                  | (6)                  | (7)                  |
|--------------------------------|-----------------------|----------------------|-----------------------|-----------------------|----------------------|----------------------|----------------------|
| Support                        | 0.0260***<br>(0.0077) | 0.0185**<br>(0.0079) | 0.0211***<br>(0.0075) | 0.0206***<br>(0.0077) | 0.0159**<br>(0.0075) | 0.0162**<br>(0.0075) | 0.0159**<br>(0.0078) |
| Customer demand                |                       |                      | 0.488***<br>(0.177)   |                       | 0.469***<br>(0.177)  | 0.415***<br>(0.155)  | 0.463***<br>(0.177)  |
| Supplier input                 |                       |                      |                       | 0.386***<br>(0.114)   | 0.386***<br>(0.112)  | 0.332***<br>(0.093)  |                      |
| Knowledge                      |                       | 0.173***<br>(0.051)  |                       |                       |                      |                      | 0.124**<br>(0.049)   |
| Input–output links             |                       |                      | all                   | all                   | all                  | top 25               | all                  |
| First-stage $\chi^2$ , support | 112.6                 | 114.0                | 113.6                 | 114.3                 | 115.4                | 115.0                | 114.8                |

*Notes:* Pooled annual regression, 2012–2019, by 2SLS with sector–technology, sector–year, and technology–year effects; the 252 sectors of the quantitative model that appear as buyers in the 2007 BEA data, 387 technologies, and 780,192 observations in every column. Adoption through each network is instrumented by the projection of the granular source shock on the same network. The first-stage statistic tests the joint relevance of the excluded instruments for support. Coefficients are not comparable across networks, whose normalizations differ. Column (6) keeps, for each receiver row, the 25 largest customers in the customer-demand network and the 25 largest suppliers in the supplier-input network. Standard errors, in parentheses, are clustered by sector and technology. \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.10$ .

implementation services are a small share of costs, the training of workers by universities enters the costs of their future employers at zero, and the know-how held by workers and practices produces few patents. Panels B and C bear this out. Manufacturing supplies 35 percent of the links of the supplier-input network and 83 percent of the links of the knowledge network; education and health supply 13 percent of the links of the estimated effective network, under one percent of inputs, and no patent citations. The entrywise correlations of the estimated effective network with the customer-demand, supplier-input, and knowledge networks are 0.029, 0.091, and 0.088, the same order as the correlations among those three networks themselves, 0.048 to 0.094 (Appendix Table D.1). The estimated effective network therefore measures a channel of its own: adoption complementarities that run through the sectors that build, staff, and teach technologies.

Table 2 holds adoption through these networks fixed in a pooled annual regression of adoption on support, with adoption through each network instrumented by the projection of the granular source shock on that network and with sector–technology, sector–year, and technology–year effects. All three networks predict adoption, consistent with previous findings. The coefficient on support remains significant at the five-percent level in every column, including when both input–output networks are included and when each network is restricted to its 25 largest links. The estimated effective network thus captures adoption complementarities not explained by production links or patent citations. Appendix D.3 describes the networks and the samples. Appendix D.4 re-estimates the loadings and the multiplier on eighteen alternative graphs built from equipment investment, common customers, market returns, occupations, patents, sales, and patent-announcement returns; seventeen give a multiplier above one and above the graph of this section on the same sectors, and the eighteenth is acyclic.

## 7 Quantifying the Dynamic Economy

Section 6 measured the empirical multiplier,  $\widehat{\text{NAM}} = \rho(\text{diag}(\hat{\beta}_s)\mathbf{G})$ . This section takes the dynamic model to the data. The quantitative economy contains the 278 sectors used in estimation and all 387 technologies, and for each technology I solve the coordination game and innovators' research choices jointly. Whether the structural standalone returns and the endogenous gaps admit distinct least and greatest equilibria is therefore an outcome of the estimation, which combines the empirical multiplier with the standalone returns. Conditional on multiplicity, hype selects the equilibrium played.

Section 7.1 summarizes the parameterization, Section 7.2 reports fit, Section 7.3 describes coordination failure, Section 7.4 presents the growth counterfactuals, and Section 7.5 evaluates welfare and policy. Appendix E collects the data construction, the calibration mapping, and the algorithm.

### 7.1 Parameterization

Table 3 summarizes the parameterization. The structural network uses the measured graph as a normalization, preserving its support, direction, and relative within-row weights. Sector mean adoption and the annual Tobit loadings determine structural standalone returns through a static mean-adoption approximation. These returns can have either sign and remain fixed during the dynamic calibration and across counterfactuals. Appendix Figure E.1 reports the distribution of standalone returns. Appendix E describes the mapping from data to parameters and provides details on the construction of the moments.

The annual discount factor is 0.96, the bargaining share is 0.50, and the outer input elasticity is 0.97. Production objects come from the 2015 producer-price Use table and BEA final expenditure.

I set the common diffusion parameter to  $\delta = 0.8619$ , the pooled annual persistence of Computat firms' revenue-TFP gaps to their sector's productivity frontier over 2010–2019. The estimate uses 34,849 consecutive firm-year transitions in 111 of the quantitative sectors.

I choose the functional form  $h_s(r) = \min\{1, \zeta_s \sqrt{r}\}$  for the arrival probability. The dynamic calibration jointly determines the adoption-cost scales  $\chi$ , success-probability scales  $\zeta$ , common innovation step  $\gamma$ , and hype transition matrix  $\Pi$ . The annual control-function Tobit estimates of Section 6 supply 272 receiver loadings (five boundary estimates are excluded). Each loading is matched to the mean structural response  $(\mathbb{E}[q_s] - 1)/\chi_s$ , and the structural multiplier is matched to the empirical  $\widehat{\text{NAM}}$  of 1.179.

Innovation targets use the breakthrough patents of Kelly et al. (2021). For breakthroughs issued in 2000–2010, I estimate the probability of receiving a first citation from another breakthrough within one to six years, using citations observed through 2016. This gives (1,470) arrival probabilities across sectors with patent origins. For each sector, the six probabilities enter the objective as overidentifying moments. Their model counterparts are the probabilities of the next innovation of the same task, measured from an innovation event. The common innovation

step is calibrated targeting the BEA-BLS all-industries TFP contribution to value-added growth averages 0.632 percentage points per year over 1998–2023. For the hype transition target, I use the 1996–2023 series of technology-adoption language in 10-K filings to classify annual hype states from smoothed hype probabilities.

**Table 3:** Quantitative parameterization

| Object                                           | Value                              | Identification or source                        |
|--------------------------------------------------|------------------------------------|-------------------------------------------------|
| <i>Panel A. Externally calibrated</i>            |                                    |                                                 |
| Discount factor $\beta$                          | 0.96                               | standard                                        |
| Bargaining share $\zeta_s$                       | 0.50                               | standard                                        |
| Outer input elasticity $\alpha$                  | 0.97                               | maintained production closure                   |
| Production objects $(\Omega, m, \tilde{\Omega})$ | 278-sector matrices                | 2015 producer-price Use before redefinitions    |
| Final shares $b_s$                               | sector-specific                    | BEA final expenditure                           |
| <i>Panel B. Estimated and preset objects</i>     |                                    |                                                 |
| Diffusion $\delta$                               | 0.862                              | external pooled Compustat gap AR(1)             |
| Adoption network $\Theta$                        |                                    | predetermined portfolios G                      |
| Standalone return $\kappa_s$                     | median $-1.132$                    | inversion of mean adoption using Tobit loadings |
| <i>Panel C. Internally calibrated</i>            |                                    |                                                 |
| Success-probability scale $\zeta_s$              | median 6.539                       | sector waiting-time distributions               |
| Adoption-cost scale $\chi_s$                     | median 4.039                       | Receiver loading $\hat{\beta}_s$                |
| Common innovation step $\gamma$                  | 1.568                              | historical aggregate TFP target                 |
| Hype transition matrix $\Pi$                     | $\pi_{00} = 0.92, \pi_{11} = 0.93$ | 10-K state-transition counts                    |

## 7.2 Model fit

Figure 8 compares the empirical receiver loadings and two-year first-successor arrival probabilities with their model counterparts. The cross-sector correlation of the receiver loadings is 0.983. The two-year arrival probabilities have a cross-sector correlation of 0.973.<sup>15</sup> Annual TFP growth is 0.628 percent in the model and 0.632 percent in the data; the model value is the calibration simulation of 1,000 periods, and the longer simulation of the counterfactuals in Section 7.4 gives 0.614 percent.

The counterfactuals use the same calibrated parameters. Appendix E.3 describes calibration and simulation.

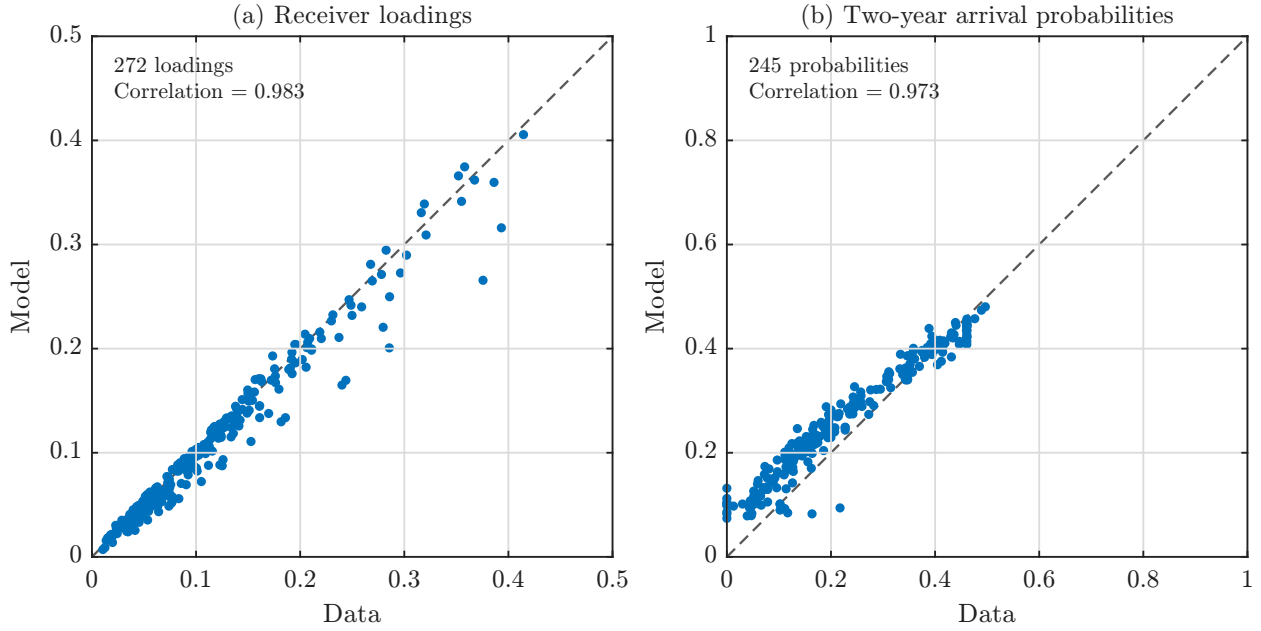
At the invariant distribution of gaps, the structural multiplier is 1.184, against 1.179 for the empirical multiplier. Since structural multiplicity also depends on standalone returns and adoption bounds, the multiplier alone summarizes only part of it: the share of simulated technology–period observations with distinct least and greatest adoption equilibria is 52.3 percent.

## 7.3 Coordination failure

On the least equilibrium, producers expect weaker adoption by the sectors that support them. Mean adoption is 0.0105 conditional on the least equilibrium and 0.693 conditional on the

<sup>15</sup>Appendix Figure E.3 reports all six arrival horizons used in calibration.

**Figure 8: MODEL FIT: 272 RECEIVER LOADINGS AND 245 TWO-YEAR ARRIVAL PROBABILITIES**



Notes: Panel (a) plots the empirical and model receiver loadings. Panel (b) plots first-successor arrival probabilities within two years for 245 sectors. The calibration uses all 1,470 arrival probabilities at horizons of one through six years. Their within-sector covariance enters the calibration objective. Appendix Figure E.3 reports all six horizons. Each dashed line is the 45-degree line.

greatest equilibrium. Lower license income in turn lowers research: mean research is 0.002 on the greatest equilibrium and 0.0001374 on the least, and the corresponding annual success probabilities are 0.265 and 0.0704.

When hype begins, adoption jumps at inherited gaps, while research and the success probability adjust to their levels in the greatest equilibrium. Subsequent successes widen the gaps in the sectors where they occur, raising the return to adoption and sustaining the initial increase. When hype ends, adoption falls and fewer innovations succeed, even as diffusion continues to raise basic productivity.

The incidence of coordination failure is heterogeneous across sectors. On the greatest equilibrium, mean adoption ranges across sectors from 0.082 to 0.946, with a median of 0.755, and the share of sectors adopting more than half of their frontier is 78.8%.

## 7.4 Growth consequences of coordination

To quantify the cost of absence of *hype*, I solve three economies at the same estimated primitives. The first uses the estimated hype process. The second selects the least equilibrium in every period, the permanent absence of hype, and the third selects the greatest equilibrium in every period, permanent hype. Under the estimated hype process, annual TFP growth is 0.614 percent (targeted). With permanent hype, growth rises to 1.007 percent. With permanent non-hype, TFP growth is 0.012 percent. Adoption affects growth through its effects on research incentives,

success probabilities, and the invariant distribution of technology gaps. In a counterfactual economy always *hyped*, the annual success probability is 0.291, compared with 0.172 under the estimated hype process. I decompose growth along the estimated path into three components. Diffusion of the frontier into basic technologies accounts for 0.286 percentage points of annual TFP growth. With unit standalone returns, ( $\kappa_s = 1$ ), and no adoption complementarities, ( $\Theta = 0$ ), innovation contributes 0.029 percentage points at the same technology gaps, holding the remaining calibrated primitives fixed. The additional innovation associated with the estimated adoption environment contributes 0.303 percentage points, or 49.1 percent of the 0.619 total.

**Table 4:** The coordination loss

|                                  | Research<br>(%/yr) | R&D intensity<br>(%) | TFP growth<br>(%/yr) |
|----------------------------------|--------------------|----------------------|----------------------|
| Permanent hype                   | 29.11              | 0.234                | 1.007                |
| Estimated hype process           | 17.20              | 0.108                | 0.614                |
| Permanent absence of hype        | 0.13               | 0.00050              | 0.012                |
| Permanent hype minus estimated   | 11.91              | 0.126                | 0.393                |
| Permanent hype minus its absence | 28.99              | 0.233                | 0.995                |

*Notes:* R&D intensity is the research bill over sales. The estimated-hype row uses the estimated hype transition matrix. The permanent absence of hype selects the least equilibrium in every period; permanent hype selects the greatest. The two permanent regimes are re-solved at the horizon and with the random numbers of the estimated hype economy.

## 7.5 Welfare and policy

I evaluate policy using the representative household's lifetime utility,  $\mathbb{E}_0 \sum_{t \geq 0} \beta^t \{\log C_t - \sum_s L_{st}^R\}$ , and report each policy's consumption equivalent: the permanent percentage change in consumption under the estimated hype process that yields the same lifetime utility. Production labor is supplied inelastically and carries no disutility, and production employment equals one in every sector under every policy. License payments, subsidies, and the lump-sum taxes used to finance them are transfers within the representative household. Welfare therefore changes through consumption and the real research labor induced by the policy.

The policies I analyze are the following. A *hype policy* makes the high adoption equilibrium played at every state while leaving private incentives unchanged; it is the benchmark for costless coordination alone. An *adoption subsidy* adds a nonnegative wedge  $\tau_s^A$  to sector  $s$ 's standalone return while leaving its productivity unchanged. An *R&D subsidy* reimburses a share of the innovator's research bill. It lowers the private cost of research, and the household bears the full disutility of the research labor employed. The adoption subsidy applies to a set of sectors and the R&D subsidy to every sector; each is set at the smallest wedge or reimbursement rate whose consumption-equivalent gain reaches the gain of the demonstration policy. Each policy experiment re-solves adoption, research, and the transition distribution from the same initial

states.

**Hype policy.** The hype policy delivers annual TFP growth of 1.007 percent. The increase is achieved through two channels: producers expect the other sectors to adopt the technologies; the innovators know that hype will persist forever, therefore they will innovate more raising growth. Historical examples include announced switchover dates and technical standards: the transition of U.S. television broadcasting to digital signals in 2009, which set a common date for broadcasters, equipment makers, and viewers; the 1987 agreement on the GSM standard, which allowed European operators and manufacturers to invest in a common mobile technology. Despite the desirability of costless policies, it is unfeasible to extend them to all the sectors across all technologies. The subsidies below are chosen to match the TFP growth.

**Targeted adoption subsidy.** I choose the sectors that receive the adoption subsidy from the first-order conditions of the cheapest wedges that move the economy from the least to the greatest equilibrium, and I evaluate the resulting subsidy in the dynamic economy. At a gap vector  $\mathbf{q}$ , let  $\underline{x}(\mathbf{q}, \tau^A)$  denote the least equilibrium with wedges  $\tau^A$ . The subsidy pays  $\lambda_s(q_s - 1) \tau_s^A \underline{x}_s$  to sector  $s$ , in units of basic productivity, where  $\lambda_s$  is the revenue share. The wedges with the lowest payment at which the least equilibrium reaches the greatest solve

$$\min_{\tau^A \geq 0} \mathcal{L} = \sum_s \lambda_s(q_s - 1) \tau_s^A \underline{x}_s(\mathbf{q}, \tau^A) - \nu \sum_s \lambda_s \left[ \min\{\underline{x}_s(\mathbf{q}, \tau^A), \bar{x}_s(\mathbf{q})\} - \bar{x}_s(\mathbf{q}) \right], \quad (32)$$

with multiplier  $\nu$ : the first term is the payment, and the bracket is minus the revenue-weighted gap that remains between the least equilibrium with wedges and the greatest equilibrium, since adoption above the greatest equilibrium earns no credit. A subsidized sector satisfies

$$\underbrace{\lambda_s(q_s - 1) \underline{x}_s}_{\text{payment on own adoption}} + \underbrace{\sum_j \lambda_j(q_j - 1) \tau_j^A \frac{\partial \underline{x}_j}{\partial \tau_s^A}}_{\text{payment on induced adoption}} = \nu \frac{q_s - 1}{\chi_s} \mathbf{c}_s^{\text{out}}(\mathbf{q}), \quad (33)$$

where  $\partial \underline{x}_j / \partial \tau_s^A$  is the response of sector  $j$ 's adoption at the least equilibrium to the wedge of sector  $s$ , and  $\mathbf{c}_s^{\text{out}}(\mathbf{q})$  is the outward centrality of sector  $s$ : the revenue-weighted sum over chains of support from  $s$ , through sectors with interior adoption, to the sectors whose adoption lies below the greatest equilibrium (Appendix E.5). A sector with  $\kappa_s + \sum_{s'} \theta_{ss'} \underline{x}_{s'} < 0$  adopts once its wedge exceeds this loss. The standalone return enters twice: a large loss raises the wedge that brings the sector to adoption, and a positive return raises the adoption the sector undertakes at the least equilibrium and with it the payment on its own adoption. Centrality enters through  $\mathbf{c}_s^{\text{out}}$ , the adoption the sector's wedge induces in the sectors it supports. Table 5 lists the ten sectors with the highest weight in the left eigenvector of the effective network per unit of the payment per unit of adoption induced at the least equilibrium, which receive the targeted adoption subsidy, next

**Table 5:** Sectors receiving the adoption subsidy and the most central sources

|    | Targeted adoption subsidy                       | Katz–Bonacich centrality                                   |
|----|-------------------------------------------------|------------------------------------------------------------|
| 1  | Cable and subscription programming (5152)       | Employment services (5613)                                 |
| 2  | Computers and peripherals (3341)                | Scientific R&D services (5417) <sup>†</sup>                |
| 3  | Semiconductors and electronic components (3344) | Colleges and universities (6113) <sup>†</sup>              |
| 4  | Data processing and hosting (5182)              | Architectural and engineering services (5413) <sup>†</sup> |
| 5  | Business support services (5614)                | General hospitals (6221) <sup>†</sup>                      |
| 6  | Other information services (5191)               | Computer systems design (5415) <sup>†</sup>                |
| 7  | Advertising and public relations (5418)         | Management and technical consulting (5416)                 |
| 8  | Software publishers (5112)                      | Other information services (5191)                          |
| 9  | Communications equipment (3342)                 | Advertising and public relations (5418)                    |
| 10 | Aerospace products and parts (3364)             | Insurance carriers (5241)                                  |

*Notes:* The first column ranks sectors by their weight in the left eigenvector of the effective network, the limit of  $\zeta^{\text{out}}$  as the restricted spectral radius approaches one (Appendix E.5), per unit of the payment per unit of adoption induced at the least equilibrium,  $\lambda_s(q_s - 1)|\kappa_s + \sum_{s'} \theta_{ss'} \underline{x}_{s'}|$ ; both are averaged over the simulated technology-period observations with distinct least and greatest adoption equilibria. The second column ranks sectors by Katz–Bonacich centrality as sources in G. NAICS codes in parentheses. <sup>†</sup>Positive standalone return,  $\kappa_s > 0$ .

to the ten sectors with the highest Katz–Bonacich centrality as sources in G.<sup>16</sup> At the common wedge of 0.431 that closes 99 percent of the gap in the static computation, every excluded sector satisfies the inequality in (33): a wedge to any of them buys less closure of the gap per unit of payment than the subsidized sectors.

A common wedge of 0.323 on these sectors reproduces the gain of the demonstration policy: its consumption-equivalent gain is 14.139 percent, annual growth is 1.008 percent, and its discounted fiscal requirement is 1.9 percent of the research bill. The wedge brings the least equilibrium close to the greatest, so adoption depends little on hype.

Table 5 shows the trade-off between centrality and the standalone return. Ranked by centrality alone, the sources are large service sectors, and half of them have positive standalone returns: they adopt at the least equilibrium, so a wedge to them pays for adoption they already undertake. Six of the ten sectors the targeted subsidy selects produce and run digital technology. Their standalone losses are moderate, so a small wedge brings them to adoption, and their adoption supports the adoption of many other sectors. These sectors are small inputs with large spillovers. Support from a sector rises with the share of its frontier it adopts, while the payment to it rises with its revenue, so the cheapest sources of support are small sectors at the center of the adoption network. The evaluated subsidy applies one wedge to the ten sectors; at the optimum the wedges differ, and (33) places the largest wedges on hardware, data centers, and internet services. The network and the responses are estimated on job postings of 2012–2019, and the sectors they select are the hardware, data, and software infrastructure of the subsequent wave of investment in artificial intelligence.

<sup>16</sup>The centrality vector is  $(I - 0.85 G^\top / \max_s \sum_{s'} G_{ss'})^{-1} \mathbf{1}$  (Ballester et al., 2006). A sector ranks high when it supports many receivers that in turn support many receivers.

**R&D subsidy.** An R&D subsidy raises innovation conditional on equilibrium selection and leaves the adoption game unchanged. The uniform R&D subsidy reaches the gain of the demonstration policy at a reimbursement rate of 0.35, at a fiscal requirement of 95 percent of the research bill. Table 6 summarizes the three policies.

**Table 6:** Welfare and industrial policy at the gain of the demonstration policy

| Policy                    | Wedge or reimbursement rate | Discounted fiscal requirement (% of research bill) | Consumption equivalent (%) | TFP growth (%/yr) |
|---------------------------|-----------------------------|----------------------------------------------------|----------------------------|-------------------|
| Estimated hype process    | —                           | —                                                  | 0                          | 0.614             |
| Demonstration policy      | —                           | 0                                                  | 14.126                     | 1.007             |
| Targeted adoption subsidy | 0.323                       | 1.9                                                | 14.139                     | 1.008             |
| Uniform R&D subsidy       | 0.35                        | 95                                                 | 15.667                     | 1.106             |

*Notes:* Consumption equivalents are relative to estimated hype and net of research-labor disutility. Subsidies are the smallest rates on the grid matching the demonstration gain. Fiscal costs are discounted annual consumption shares, benchmarked against the hype research bill (0.187 percent of consumption). The adoption subsidy targets the ten sectors in Table 5. Payments are lump-sum financed, and output is valued at standalone returns. Monte Carlo standard errors are 0.628, 0.624, and 0.432 percentage points, respectively. See Appendix E.5.

## 8 Conclusion

This paper argues that a short term failure in adopting frontiers leads to long-run reduction in growth. Namely, when the return to using a frontier depends on adoption in complementary sectors, multiplicity arises. Which equilibrium is played then changes the market innovators expect to face, and hence the incentive to advance the frontier.

This paper develops and measures this feedback. The Network Adoption Multiplier summarizes the strength of mutual support in adoption and determines when coordination failure can arise. I measure the adoption network from the technology content of U.S. job postings and find a Network Adoption Multiplier above one, with adoption that begins once support is large. Embedding these estimates in the dynamic model yields distinct low and high adoption equilibria in 52.3% of simulated technology-period observations. The consequence of coordination failures matters for innovation: low adoption lowers the returns from innovating, hence lowers research. These effects accumulate into long-run growth. Under the estimated hype process, annual TFP growth is 0.614 percent; permanently selecting the high adoption equilibrium gives 1.007 percent. There are three sets of policies. Firstly, a *hype* policy, where, through expectations, the planner can make all the sectors coordinate on high adoption. Second, adoption subsidies targeted to sectors that have small standalone losses, but high out-degree complementarities. Third, research subsidies. Subsidizing research allows to increase growth by affecting the productivity gap, without resolving the coordination failure, hence preserving multiplicity.

The central implication is that the use and creation of technology cannot generally be separated. An economy can possess a productive frontier yet fail to coordinate on using it. Furthermore, when innovation responds to adoption, that failure also reduces the incentive to

move the frontier forward. Coordination therefore affects long-run growth through which future technologies are worth creating.

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# Appendices

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## A Proofs

### A.1 Proof of Lemma 1

*Proof of Lemma 1.* Fix  $\mathbf{q}$  and let  $T$  be the best-response map,  $T_s(\mathbf{x}) \equiv \text{BR}_s(\mathbf{x}_{-s}, q_s)$  for every  $s$ ; its fixed points are the adoption equilibria. The componentwise ordered set  $[0, 1]^S$  is a complete lattice. Since  $\hat{\Theta}(\mathbf{q}) \geq 0$ ,  $\mathbf{x} \leq \mathbf{y}$  implies

$$\hat{\kappa} + \hat{\Theta}\mathbf{x} \leq \hat{\kappa} + \hat{\Theta}\mathbf{y}.$$

Coordinatewise truncation preserves order, so  $T$  is isotone. By Tarski's fixed-point theorem the set of fixed points is nonempty and a complete lattice. For the low adoption equilibrium set  $\mathbf{x}^0 = \mathbf{0}$  and  $\mathbf{x}^{n+1} = T(\mathbf{x}^n)$ . Since  $\mathbf{0} \leq T(\mathbf{0})$ , isotonicity gives  $\mathbf{x}^{n+1} \geq \mathbf{x}^n$  by induction. The sequence is bounded above by  $\mathbf{1}$ , so it converges componentwise to some  $\mathbf{x}^\infty \in [0, 1]^S$ . By continuity of  $T$ ,  $\mathbf{x}^\infty = T(\mathbf{x}^\infty)$ . For any fixed point  $\mathbf{y}$ ,  $\mathbf{x}^0 \leq \mathbf{y}$  so  $\mathbf{x}^n \leq T^n(\mathbf{y}) = \mathbf{y}$ . Hence  $\mathbf{x}^\infty \leq \mathbf{y}$ , proving that this limit is the least fixed point.

The same argument from  $\mathbf{1}$ , with  $T(\mathbf{1}) \leq \mathbf{1}$  and a nonincreasing sequence, gives the greatest fixed point.  $\square$

### A.2 Proof of Lemma 2

*Proof of Lemma 2.* Fix  $\mathbf{q}$ . (i)  $\mathbf{0}$  is an equilibrium iff  $T_s(\mathbf{0}) = [\hat{\kappa}_s]_{[0,1]} = 0$  for every  $s$ , that is iff  $\hat{\kappa} \leq \mathbf{0}$ ; being the smallest profile it is then the least equilibrium. If  $\hat{\kappa}_s > 0$ ,  $T_s(\mathbf{x}) \geq \min\{\hat{\kappa}_s, 1\} > 0$  at every profile, so  $x_s > 0$  in every equilibrium. (ii)  $\mathbf{1}$  is an equilibrium iff  $T_s(\mathbf{1}) = [\hat{\kappa}_s + \sum_{s'} \hat{\theta}_{ss'}]_{[0,1]} = 1$  for every  $s$ , that is iff  $\hat{\kappa} + \hat{\Theta}\mathbf{1} \geq \mathbf{1}$ ; being the largest profile it is then the greatest equilibrium. (iii) If  $\hat{\kappa} \geq \mathbf{1}$  then  $T(\mathbf{x}) = \mathbf{1}$  for every  $\mathbf{x}$ , so  $\mathbf{1}$  is the unique fixed point.  $\square$

### A.3 Proof of Theorem 1

*Proof of Theorem 1.* Fix  $\mathbf{q}$  and write  $\hat{\Theta} = \hat{\Theta}(\mathbf{q})$ . Suppose  $\rho(\hat{\Theta}) < 1$ .

**Contraction.** The Neumann series converges, and

$$\mathbf{w} = (I - \hat{\Theta})^{-1}\mathbf{1} = \sum_{n=0}^{\infty} \hat{\Theta}^n \mathbf{1}$$

is finite and satisfies  $\mathbf{w} \geq \mathbf{1} \gg \mathbf{0}$ . Moreover,

$$\hat{\Theta}\mathbf{w} = \mathbf{w} - \mathbf{1}.$$

Define the weighted sup norm  $\|\mathbf{x}\|_{\mathbf{w}} = \max_s |x_s|/w_s$ . For any  $\mathbf{x}, \mathbf{y} \in [0, 1]^S$ , nonnegativity of  $\hat{\Theta}$  gives

$$|[\hat{\Theta}(\mathbf{x} - \mathbf{y})]_s| \leq \sum_{s'} \hat{\theta}_{ss'} |x_{s'} - y_{s'}| \leq (\hat{\Theta}\mathbf{w})_s \|\mathbf{x} - \mathbf{y}\|_{\mathbf{w}}.$$

Projection onto a closed interval is one-Lipschitz. Apply this fact coordinatewise, divide by  $w_s$ , and maximize over  $s$ :

$$\|T(\mathbf{x}) - T(\mathbf{y})\|_{\mathbf{w}} \leq \max_s \frac{(\hat{\Theta}\mathbf{w})_s}{w_s} \|\mathbf{x} - \mathbf{y}\|_{\mathbf{w}} = \max_s \left(1 - \frac{1}{w_s}\right) \|\mathbf{x} - \mathbf{y}\|_{\mathbf{w}},$$

and the coefficient is below one. Hence  $T$  is a contraction: it has a unique fixed point, reached by iteration from every  $\mathbf{x}^0 \in [0, 1]^S$ .

**Interior formula.** If the unique equilibrium is interior, no truncation constraint binds, so

$$\mathbf{x}^* = \hat{\kappa} + \hat{\Theta}\mathbf{x}^*.$$

Because  $I - \hat{\Theta}$  is invertible,

$$\mathbf{x}^* = (I - \hat{\Theta})^{-1} \hat{\kappa}.$$

□

#### A.4 Proof of Theorem 2

*Proof of Theorem 2.* Fix  $\mathbf{q}$  and write  $\hat{\Theta} = \hat{\Theta}(\mathbf{q})$  and  $\text{NAM} = \rho(\hat{\Theta})$ .

*Part (i), necessity.* If  $\text{NAM} < 1$ , Theorem 1 gives uniqueness. It remains to rule out multiplicity at  $\text{NAM} = 1$  under the strict inequalities  $\hat{\kappa} \ll \mathbf{0}$ .

Suppose, toward a contradiction, that there is a nonzero equilibrium  $\mathbf{x}$ . Let  $\mathcal{A} = \{s : x_s > 0\}$ . For every  $s \in \mathcal{A}$ , the lower truncation does not bind, and therefore

$$x_s \leq \hat{\kappa}_s + \sum_{s' \in \mathcal{A}} \hat{\theta}_{ss'} x_{s'}.$$

Since  $\hat{\kappa}_s < 0$ ,

$$\mathbf{x}_{\mathcal{A}} < \hat{\Theta}_{\mathcal{A}\mathcal{A}} \mathbf{x}_{\mathcal{A}} \quad \text{componentwise.}$$

Because  $\mathbf{x}_{\mathcal{A}} \gg \mathbf{0}$ ,  $\rho(\hat{\Theta}_{\mathcal{A}\mathcal{A}}) > 1$ . But a principal submatrix of a nonnegative matrix cannot have spectral radius larger than that of the full matrix, so

$$1 < \rho(\hat{\Theta}_{\mathcal{A}\mathcal{A}}) \leq \rho(\hat{\Theta}) = 1,$$

a contradiction. Hence  $\mathbf{0}$  is the only equilibrium when  $\text{NAM} \leq 1$ . Coordination failure therefore requires  $\text{NAM} > 1$ .

*Part (i), sufficiency.* Since  $\hat{\kappa} \ll \mathbf{0}$ ,

$$T(\mathbf{0}) = [\hat{\kappa}]_{[0,1]} = \mathbf{0},$$

so  $\underline{\mathbf{x}} = \mathbf{0}$ .

Let  $\mathbf{c}$  be as in (17):  $\mathbf{c} \geq \mathbf{0}$ ,  $\hat{\Theta}\mathbf{c} = \text{NAM} \mathbf{c}$ , and  $\max_s c_s = 1$ ; a nonnegative eigenvector associated

with the spectral radius exists for every nonnegative matrix (Perron–Frobenius). For every  $s$  with  $\mathbf{c}_s > 0$ , the condition  $\widehat{\kappa}_s \geq \underline{\kappa}_s = -(\text{NAM} - 1)\mathbf{c}_s$  gives

$$\widehat{\kappa}_s + \sum_{s'} \widehat{\theta}_{ss'} \mathbf{c}_{s'} = \widehat{\kappa}_s + \text{NAM} \mathbf{c}_s \geq \mathbf{c}_s.$$

and truncation to  $[0, 1]$  preserves the inequality since  $0 < \mathbf{c}_s \leq 1$ . For every  $s$  with  $\mathbf{c}_s = 0$ ,  $T_s(\mathbf{c}) = [\widehat{\kappa}_s]_{[0,1]} = 0 = \mathbf{c}_s$ . Hence  $T(\mathbf{c}) \geq \mathbf{c}$ . Monotone iteration from  $\mathbf{c}$  converges to a fixed point  $\mathbf{x}^* \geq \mathbf{c}$ , and therefore

$$\bar{\mathbf{x}} \geq \mathbf{x}^* \geq \mathbf{c}.$$

Since  $\max_s \mathbf{c}_s = 1$ ,  $\bar{\mathbf{x}} \neq \mathbf{0} = \underline{\mathbf{x}}$ .

*Part (ii).* Theorem 1 implies that  $\text{NAM} < 1$  gives a unique equilibrium for every vector of standalone returns. Hence

$$\underline{\mathbf{x}} \neq \bar{\mathbf{x}} \implies \text{NAM} \geq 1.$$

□

*Proof of the fee ordering in Theorem 2.* Let  $b \in \{L, H\}$  denote the least and greatest adoption equilibria. For sector  $s$ , define

$$v_s^b \equiv (q_s - 1) \left( \kappa_s + \sum_{s' \neq s} \theta_{ss'} x_{s'}^b \right).$$

Conditional on the other sectors' adoption, pair  $s$  chooses  $x$  to maximize

$$v_s^b x - \frac{\chi_s}{2} x^2 \quad \text{over } x \in [0, 1].$$

Let

$$\mathcal{V}_s(v) \equiv \max_{x \in [0,1]} \left\{ vx - \frac{\chi_s}{2} x^2 \right\}.$$

The maximizer is  $[v/\chi_s]_{[0,1]}$ , and

$$\mathcal{V}_s(v) = \begin{cases} 0, & v \leq 0, \\ v^2/(2\chi_s), & 0 < v < \chi_s, \\ v - \chi_s/2, & v \geq \chi_s. \end{cases} \quad (34)$$

Hence  $\mathcal{V}_s$  is nondecreasing on  $\mathbb{R}$  and strictly increasing on  $(0, \infty)$ , and the productivity ratio chosen by the pair is

$$a_s^b \equiv \frac{A_s^b}{A_s^B} = 1 + \mathcal{V}_s(v_s^b).$$

Since  $\mathbf{x}^H \geq \mathbf{x}^L$ ,  $\theta_{ss'} \geq 0$ , and  $q_s - 1 \geq 0$ ,

$$v_s^H \geq v_s^L \quad \text{for every } s.$$

It follows that

$$a_s^H \geq a_s^L \quad \text{for every } s.$$

Moreover, if  $x_s^H > x_s^L$ , then the corresponding unconstrained return must be strictly larger:  $v_s^H > v_s^L$ , with  $v_s^H > 0$ . Therefore

$$a_s^H > a_s^L.$$

Thus every sector whose own adoption differs across the two equilibria obtains a strictly larger productivity ratio on the greatest equilibrium.

Per unit of revenue, from (18),

$$f_s^b \equiv \frac{\phi_s^b}{R_s^b} = \zeta_s \left( 1 - \frac{\alpha}{\mu_s} \right) \left[ 1 - (a_s^b)^{-\frac{1-\alpha}{\mu_s-\alpha}} \right].$$

This expression is nondecreasing in  $a_s^b$ , and strictly increasing for  $a_s^b > 1$ . Hence

$$f_s^H \geq f_s^L \quad \text{for every } s,$$

with strict inequality whenever  $x_s^H > x_s^L$ .

For the level fee, sectoral revenue satisfies  $R_s^b = \lambda_s C^b$ . The greatest adoption equilibrium weakly raises every sector's effective productivity relative to the least, so the production block implies  $C^H \geq C^L$ , and therefore  $R_s^H \geq R_s^L$  for every  $s$ . Since  $\phi_s^b = R_s^b f_s^b$ ,

$$\phi_s^H \geq \phi_s^L \quad \text{for every } s,$$

with strict inequality whenever  $x_s^H > x_s^L$ .

Under the conditions of part (i) for  $\bar{\mathbf{x}} \geq \mathbf{c}$ ,  $\mathbf{x} = \mathbf{0}$  and  $\bar{\mathbf{x}} \geq \mathbf{c}$ , so  $\bar{x}_s \geq c_s > 0 = \underline{x}_s$  in every sector with  $c_s > 0$ , and the strict comparison holds in those sectors.  $\square$

## A.5 Proof of Lemma 3

*Proof of Lemma 3.* Three facts about the flows in (13) give (20). First, by (18) and the constant profit share of revenue,  $\phi_{st} = R_{st} f_s(\mathbf{q}_t, z_t)$  with  $f_s$  as in (19), and  $f_s$  depends on the state only through  $(\mathbf{q}_t, z_t)$  because the selected profile does. Second,  $w_{st}^R = C_t$  and  $L_{st}^R = \lambda_s r_{st}$  give  $w_{st}^R L_{st}^R = R_{st} r_{st}$ . Third,  $R_{s,t+1}/R_{st} = C_{t+1}/C_t$ , so under the guess  $V_s(\mathbf{A}_{t+1}^B, \mathbf{q}_{t+1}, z_{t+1}) = R_{s,t+1} v_s(\mathbf{q}_{t+1}, z_{t+1})$  the discounted continuation value is  $\beta R_{st} v_s(\mathbf{q}_{t+1}, z_{t+1})$ . Substituting the three into (13) and dividing by  $R_{st}$  gives (20), the choice of  $L_{st}^R$  being the choice of  $r_{st}$ .

Given the other sectors' policies and the selection rule, the right-hand side of (20) defines an operator  $\mathcal{T}$  on bounded functions on  $\mathcal{Q} \times \{0, 1\}$ . By (37),  $0 \leq f_s \leq \bar{f}_s$ , so  $\beta \inf v \leq \mathcal{T}v \leq$

$\bar{f}_s + \beta \sup v$  (the lower bound at  $r = 0$ ) and  $\mathcal{T}$  maps bounded functions to bounded functions;  $\mathcal{T}$  is monotone and  $\mathcal{T}(v + a) = \mathcal{T}v + \beta a$  for every constant  $a$ , so it is a  $\beta$ -contraction and has a unique bounded fixed point,  $v_s \in [0, \bar{f}_s / (1 - \beta)]$ . Then  $V_s = R_{st}v_s$  solves (13).

For the maximizer, let  $\Delta_s(\mathbf{q}, z)$  be the success premium,  $\beta$  times the expected difference between the continuation value at  $\gamma_s$  and at  $q_s^{\delta_s}$ . The objective in  $r$  is  $-r + h_s(r)\Delta_s$  up to a constant. If  $\Delta_s \leq 0$  it is strictly decreasing and the maximizer is  $r = 0$ . If  $\Delta_s > 0$  it is strictly concave wherever  $h_s(r) < 1$  and strictly decreasing beyond it, so the maximizer is unique, and  $\Delta_s \leq \beta \bar{f}_s / (1 - \beta)$  gives the bound (38).  $\square$

## A.6 Proof of Theorem 3

*Proof.* Since  $\kappa_s < 0$  for every sector, the least adoption equilibrium is  $\mathbf{0}$  at every gap vector, so  $f_s(\mathbf{q}, 0) = 0$ . On the greatest equilibrium,  $f_s(\mathbf{q}, 1) \geq 0$ . Moreover,  $f_s(\mathbf{q}, 1)$  is nondecreasing in  $q_s$  (Corollary C.2).

By assumption, the law of  $\mathbf{q}'_{-s}$  given  $\mathbf{q}$  is the same under both values of the current public sunspot and does not depend on  $q_s$ .

*Part (i).* Persistence implies

$$\pi_{11} - \pi_{01} = \pi_{00} + \pi_{11} - 1 > 0.$$

The flow difference between hype and no hype is nonnegative. Starting from the zero function, value iteration preserves this ordering: whenever  $v_s(\mathbf{q}, 1) \geq v_s(\mathbf{q}, 0)$ , persistence assigns more weight to the higher continuation value following  $z = 1$  than following  $z = 0$ , while the current fee is also weakly higher. Since the Bellman operator is a contraction, the ordering carries to its fixed point. Hence

$$v_s(\mathbf{q}, 1) \geq v_s(\mathbf{q}, 0)$$

for every sector  $s$  and every  $\mathbf{q} \in \mathcal{Q}$ .

*Part (ii).* At  $\delta_s = 0$ , a failure closes sector  $s$ 's gap to one, while a success resets it to  $\gamma_s$ . Because the law of  $\mathbf{q}'_{-s}$  does not depend on  $q_s$ , sector  $s$ 's own gap affects its value only through the current fee; since the fee difference is nondecreasing in  $q_s$ ,

$$v_s(\gamma_s, \mathbf{q}_{-s}, 1) - v_s(\gamma_s, \mathbf{q}_{-s}, 0) \geq v_s(1, \mathbf{q}_{-s}, 1) - v_s(1, \mathbf{q}_{-s}, 0).$$

It follows that the difference between the success premia under hype and no hype is

$$\beta(\pi_{11} - \pi_{01}) \mathbb{E} \left[ \begin{array}{c} v_s(\gamma_s, \mathbf{q}'_{-s}, 1) - v_s(\gamma_s, \mathbf{q}'_{-s}, 0) \\ - v_s(1, \mathbf{q}'_{-s}, 1) + v_s(1, \mathbf{q}'_{-s}, 0) \end{array} \middle| \mathbf{q} \right] \geq 0.$$

The research objective is  $-r + h_s(r)\Delta$ , up to terms independent of  $r$ , where  $\Delta$  is the success premium. Since  $h_s$  is increasing, the objective has increasing differences in research and the

success premium. Therefore

$$r_s^*(\mathbf{q}, 1) \geq r_s^*(\mathbf{q}, 0)$$

for every  $\mathbf{q} \in \mathcal{Q}$ .

Finally, suppose the inequality at  $\delta_s = 0$  is strict with a uniform margin,  $\inf_{\mathbf{q} \in \mathcal{Q}} [r_s^*(\mathbf{q}, 1) - r_s^*(\mathbf{q}, 0)] > 0$ . As  $\delta_s \rightarrow 0$ , the failure state  $q_s^{\delta_s} \rightarrow 1$  uniformly on  $\mathcal{Q}$ , and the fee at that state vanishes uniformly: by (7) and (16), the productivity ratio satisfies  $a_s - 1 \leq (q_s - 1) \sum_{s'} \theta_{ss'}$  whatever the adoption profile, so  $\sup_{\mathbf{q}_{-s}, z} f_s(q_s^{\delta_s}, \mathbf{q}_{-s}, z) \rightarrow 0$ . Since the transition law of the other sectors' gaps is held fixed, and  $v_s^0$  depends on the own gap only through the current fee, applying the Bellman operator at  $\delta_s$  to  $v_s^0$  moves it, in sup norm, by at most  $\beta$  times this fee bound; as the operator is a  $\beta$ -contraction,  $\|v_s^{\delta_s} - v_s^0\|_\infty \leq \beta \sup f / (1 - \beta) \rightarrow 0$ , hence the success premium converges uniformly, and so does the research policy, which is a continuous function of the premium. A uniform limit preserves a uniform strict margin, so the inequality holds for all sufficiently small  $\delta_s > 0$ .  $\square$

## A.7 Proof of Proposition 1

*Proof. Part (i).* At  $\mathbf{q}_0$ , the conditions of Theorem 2(i) hold. Since all standalone returns are negative,  $\mathbf{0}$  is the least equilibrium. Since  $\max_s \mathbf{c}_s = 1$  and  $\widehat{\kappa}_s(q_{s0}) < 0$ , the condition  $\widehat{\kappa}_s(q_{s0}) \geq \underline{\kappa}_s(\mathbf{q}_0) = -(\text{NAM}(\mathbf{q}_0) - 1)\mathbf{c}_s$  at a sector with  $\mathbf{c}_s = 1$  requires  $\text{NAM}(\mathbf{q}_0) > 1$ , and Theorem 2(i) gives

$$\bar{\mathbf{x}}(\mathbf{q}_0) \geq \mathbf{c} \neq \mathbf{0}.$$

Hence the least and greatest equilibria are distinct.

*Part (ii).* The effective standalone return of sector  $s$  is

$$\widehat{\kappa}_s(q_s) = \frac{q_s - 1}{\chi_s} \kappa_s.$$

Since  $\widehat{\kappa}_s(q_{s0}) < 0$ ,  $q_{s0} > 1$  implies  $\kappa_s < 0$ . Thus  $\widehat{\kappa}_s(q_s) \leq 0$  at every  $q_s \in [1, \gamma_s]$ , and  $\mathbf{0}$  is the least adoption equilibrium at every gap vector.

Since  $z_0 = 0$  and  $\pi_{00} = 1$ ,  $z_t = 0$  at every date, so the least equilibrium is selected at every date. Hence adoption is zero and the license fee satisfies

$$f_s(\mathbf{q}, 0) = 0$$

for every sector and every state. The Bellman equation then has  $v_s(\mathbf{q}, 0) = 0$  as its unique bounded solution, and the research objective is maximized uniquely at  $r_s = 0$ . No innovation succeeds. Equation (15) therefore gives

$$q_{s,t+1} = q_{st}^{\delta_s}, \quad q_{st} = q_{s0}^{\delta_s^t} \longrightarrow 1.$$

Part (iii). Every entry in row  $s$  of the effective network is

$$\hat{\theta}_{ss'}(q_{st}) = \frac{q_{st} - 1}{\chi_s} \theta_{ss'}.$$

Since  $q_{st}$  decreases monotonically to one,  $\hat{\Theta}(\mathbf{q}_t)$  decreases entrywise. Monotonicity of the spectral radius for nonnegative matrices therefore implies that  $\text{NAM}(\mathbf{q}_t)$  is nonincreasing.

Moreover,

$$\text{NAM}(\mathbf{q}_t) \leq \max_s \frac{q_{s0}^{\delta_s^t} - 1}{\chi_s} \sum_{s'} \theta_{ss'},$$

where the right-hand side converges to zero. Hence there is a finite date

$$t^* = \min \left\{ t \in \mathbb{N} : \max_s \frac{(q_{s0}^{\delta_s^t} - 1) \sum_{s'} \theta_{ss'}}{\chi_s} < 1 \right\}$$

such that  $\text{NAM}(\mathbf{q}_t) < 1$  for every  $t \geq t^*$ .

Part (iv). For every state with  $\text{NAM}(\mathbf{q}) < 1$ , Theorem 1 gives a unique adoption equilibrium. The least and greatest equilibria therefore coincide. Since gaps continue to narrow while hype is absent, the effective network continues to weaken and the multiplier remains below one after  $t^*$ . A change in the public sunspot at any such gap vector leaves adoption and license fees unchanged until an innovation success widens a gap enough to restore the possibility of distinct equilibria.  $\square$

## A.8 Proof of Proposition 2

Write  $h_s(\mathbf{q}, z) = h_s(r_s(\mathbf{q}, z))$  for sector  $s$ 's success probability under the equilibrium research policy, so that  $h_s(r_{st}) = h_s(\mathbf{q}_t, z_t)$ . By (15) and the conditional independence of innovation outcomes, given the history up to  $t$  the coordinates of  $\mathbf{q}_{t+1}$  are independent, with  $q_{s,t+1} = \gamma_s$  with probability  $h_s(\mathbf{q}_t, z_t)$  and  $q_{s,t+1} = q_{st}^{\delta_s}$  otherwise. The sunspot is exogenous:  $z_{t+1}$  is drawn from  $\Pi$  given  $z_t$ , independently of innovation outcomes.

*Proof.* Part (i). Fix a state  $(\mathbf{q}, z)$  with  $h_s(\mathbf{q}, z) \geq \tau_s(q_s)$  for every  $s$ . Given the history, the distribution of  $\mathbf{q}_{t+1}$  depends on the past only through  $(\mathbf{q}_t, z_t)$ , so it suffices to show  $\mathbb{E}[\text{NAM}(\mathbf{q}_{t+1}) \mid \mathbf{q}_t = \mathbf{q}, z_t = z] \geq \text{NAM}(\mathbf{q})$ . If  $\text{NAM}(\mathbf{q}) = 0$  there is nothing to prove, so let  $\text{NAM}(\mathbf{q}) > 0$ . A sector with  $q_s = 1$  has a zero row in  $\hat{\Theta}(\mathbf{q})$  and a nonnegative row in  $\hat{\Theta}(\mathbf{q}_{t+1})$ ; since the spectral radius of a nonnegative matrix is nondecreasing in each entry, it suffices to prove the inequality with such rows kept at zero next period, which is the same argument on the remaining coordinates. Hence assume  $q_s > 1$  for every  $s$ . Let  $\ell \in \mathbb{R}^S$  have coordinates  $\ell_s = \log[(q_s - 1)/\chi_s]$ , so that  $\hat{\Theta}(\mathbf{q}) = \text{diag}(e^\ell)\Theta$ , and define

$$f(\mathbf{m}) = \log \rho(\text{diag}(e^{\mathbf{m}})\Theta), \quad \mathbf{m} \in \mathbb{R}^S,$$

with  $f = -\infty$  where  $\rho = 0$ , so that  $\text{NAM}(\mathbf{q}) = e^{f(\ell)}$ .

$f$  is convex and nondecreasing. Along any line in  $\mathbb{R}^S$ , each entry of  $\text{diag}(e^{\mathbf{m}})\Theta$  is either identically zero or a log-affine function of the line parameter, hence log-convex. By [Kingman \(1961\)](#), the spectral radius of a nonnegative matrix whose entries are log-convex functions of a real parameter is itself log-convex in that parameter. Hence  $f$  is convex along every line, that is, convex on  $\mathbb{R}^S$ . Since the spectral radius of a nonnegative matrix is nondecreasing in each entry,  $f$  is nondecreasing in each coordinate.

The expected log gaps do not fall. Let  $\ell'$  be the random vector with  $\ell'_s = \log[(\gamma_s - 1)/\chi_s]$  with probability  $h_s(\mathbf{q}, z)$  and  $\ell'_s = \log[(q_s^{\delta_s} - 1)/\chi_s]$  otherwise, independently across  $s$ , so that  $\widehat{\Theta}(\mathbf{q}_{t+1}) = \text{diag}(e^{\ell'})\Theta$  in distribution given  $(\mathbf{q}_t, z_t) = (\mathbf{q}, z)$ . If  $\delta_s = 0$ , the hypothesis gives  $h_s(\mathbf{q}, z) = 1$ , so  $\ell'_s = \log[(\gamma_s - 1)/\chi_s] \geq \ell_s$  with probability one. If  $\delta_s \in (0, 1)$ , then

$$\mathbb{E}\ell'_s - \ell_s = h_s(\mathbf{q}, z) \log \frac{\gamma_s - 1}{q_s^{\delta_s} - 1} - \log \frac{q_s - 1}{q_s^{\delta_s} - 1},$$

and since  $1 < q_s^{\delta_s} < q_s \leq \gamma_s$ , both logarithms are positive, so the right-hand side is nonnegative exactly when  $h_s(\mathbf{q}, z) \geq \tau_s(q_s)$ . In either case  $\mathbb{E}\ell' \geq \ell$  coordinatewise, and  $\ell'$  is almost surely finite.

The vector  $\ell'$  takes finitely many values, so Jensen's inequality applies to the convex function  $f$ , and monotonicity then gives  $\mathbb{E}f(\ell') \geq f(\mathbb{E}\ell') \geq f(\ell)$ . Since  $\exp$  is convex,

$$\mathbb{E}[\text{NAM}(\mathbf{q}_{t+1}) \mid \mathbf{q}_t = \mathbf{q}, z_t = z] = \mathbb{E}e^{f(\ell')} \geq e^{\mathbb{E}f(\ell')} \geq e^{f(\ell)} = \text{NAM}(\mathbf{q}).$$

*Part (ii).* Write  $\mathcal{R}$  for the region of multiple equilibria.

*Step 1:  $\mathcal{R}$  is closed under wider gaps and contains  $\gamma$ .* Let  $T_{\mathbf{q}} : [0, 1]^S \rightarrow [0, 1]^S$  be the best-response map (16) at  $\mathbf{q}$ . By Lemma 1,  $\bar{\mathbf{x}}(\mathbf{q}) = \lim_n T_{\mathbf{q}}^n(\mathbf{1})$ . Let  $\mathbf{q} \leq \mathbf{q}'$  coordinatewise and fix  $\mathbf{x} \in [0, 1]^S$  and  $s$ . If  $\kappa_s + \sum_{s' \neq s} \theta_{ss'} x_{s'} \geq 0$ , then multiplying it by  $(q_s - 1)/\chi_s$  gives a smaller number than multiplying it by  $(q'_s - 1)/\chi_s$ , and since  $y \mapsto \max\{\min\{y, 1\}, 0\}$  is nondecreasing,  $T_{\mathbf{q}}(\mathbf{x})_s \leq T_{\mathbf{q}'}(\mathbf{x})_s$ . If it is negative, both products are nonpositive and both best responses equal zero. Hence  $T_{\mathbf{q}} \leq T_{\mathbf{q}'}$  pointwise. Since  $\theta_{ss'} \geq 0$ , each  $T_{\mathbf{q}'}$  is nondecreasing in  $\mathbf{x}$ , and induction gives  $T_{\mathbf{q}}^n(\mathbf{1}) \leq T_{\mathbf{q}'}^n(\mathbf{1})$  for every  $n$ : the case  $n = 0$  is trivial, and  $T_{\mathbf{q}}^{n+1}(\mathbf{1}) = T_{\mathbf{q}}(T_{\mathbf{q}}^n(\mathbf{1})) \leq T_{\mathbf{q}'}(T_{\mathbf{q}}^n(\mathbf{1})) \leq T_{\mathbf{q}'}(T_{\mathbf{q}'}^n(\mathbf{1}))$ . Taking limits,  $\bar{\mathbf{x}}(\mathbf{q}) \leq \bar{\mathbf{x}}(\mathbf{q}')$ . With negative standalone returns,  $\mathbf{0}$  is a fixed point of every  $T_{\mathbf{q}}$  and hence the least equilibrium, so  $\mathcal{R} = \{\mathbf{q} : \bar{\mathbf{x}}(\mathbf{q}) \neq \mathbf{0}\}$ , and  $\mathbf{q} \in \mathcal{R}, \mathbf{q}' \geq \mathbf{q}$  imply  $\mathbf{q}' \in \mathcal{R}$ . Since  $\mathbf{q} \leq \gamma$  for every  $\mathbf{q} \in \mathcal{Q}$  and  $\mathcal{R} \neq \emptyset, \gamma \in \mathcal{R}$ .

*Step 2: re-entry in finite time.* Let  $\underline{h} > 0$  be the lower bound on success probabilities under hype outside  $\mathcal{R}$ , and condition on the entire path of the sunspot, which is independent of innovation outcomes. Under either recurrence condition, the number of hype periods before  $t$  grows without bound almost surely: if  $z_0 = 1$  and  $\pi_{11} = 1$ , then  $z_t = 1$  at every date; if  $\pi_{01} > 0$  and  $\pi_{11} = 1$  the chain is absorbed at hype, and otherwise it is irreducible on  $\{0, 1\}$ . At any date  $t$  with  $\mathbf{q}_t \notin \mathcal{R}$  and  $z_t = 1$ , all  $S$  sectors succeed with conditional probability  $\prod_s h_s(\mathbf{q}_t, 1) \geq \underline{h}^S$ , and then  $\mathbf{q}_{t+1} = \gamma \in \mathcal{R}$ . Hence, given the sunspot path, the conditional probability that  $\mathcal{R}$  has not been

entered by  $t + 1$  is at most  $1 - \underline{h}^S$  times the indicator that it has not been entered by  $t$  when  $z_t = 1$ , and at most that indicator when  $z_t = 0$ . Iterating, the probability that  $\mathcal{R}$  has not been entered by  $t$  is at most  $(1 - \underline{h}^S)^{N_t}$ , where  $N_t$  is the number of hype periods before  $t$ . Integrating over the sunspot path, the unconditional probability is at most  $\mathbb{E}[(1 - \underline{h}^S)^{N_t}]$ . Since  $N_t \rightarrow \infty$  almost surely, the bound tends to zero by dominated convergence:  $\mathcal{R}$  is entered in finite time with probability one.

*Step 3: a single success.* Let  $s$  be a sector whose success alone would move  $\mathbf{q}_t$  into  $\mathcal{R}$ , and suppose it succeeds at  $t$ , which happens with probability  $h_s(r_{st})$ . Whatever the other sectors do, every coordinate of  $\mathbf{q}_{t+1}$  is at least the corresponding coordinate of the vector obtained from  $\mathbf{q}_t$  by resetting  $q_{st}$  to  $\gamma_s$  and letting every other gap diffuse one period, because each other coordinate is either  $\gamma_{s'}$  or  $q_{s't}^{\delta_{s't}}$ . That vector lies in  $\mathcal{R}$  by assumption, so  $\mathbf{q}_{t+1} \in \mathcal{R}$  by Step 1.  $\square$

## A.9 Proof of Theorem 4

Notation as in Appendix B.5.

*Proof of Theorem 4.* Since  $x_s = 0$  is feasible,  $0 \leq \psi_s \leq K_s$  on the compact state space for some finite  $K_s$ . Under an invariant law  $\mathbb{P}^*$ , with  $\mathbb{E}^*$  the expectation under it, the states at  $t$  and  $t + 1$  have the same distribution, so  $\mathbb{E}^*[\psi_s(\mathbf{q}_{t+1}, z_{t+1}) - \psi_s(\mathbf{q}_t, z_t)] = 0$  (a nonzero mean drift  $c$  would give  $\mathbb{E}[\psi_s(\mathbf{q}_T, z_T)] = \mathbb{E}[\psi_s(\mathbf{q}_0, z_0)] + cT$ , violating the bound). Taking  $\mathbb{P}^*$ -expectations in (21) therefore leaves only the basic term,  $\mathbb{E}^*[\Delta \log A_{s,t+1}^B] = \int g_s^B d\mathbb{P}^*$ , which is (22). If  $\mathbb{P}^*$  is ergodic, one-period growth is bounded and the ergodic theorem gives the almost-sure time average.  $\square$

## A.10 Proof of Theorem 5

Notation as in Appendix B.5.

*Proof of Theorem 5.* At a common gap vector  $\mathbf{q}$ , the conditional expectation of (21) under regime  $b$  is  $(1 - \alpha) \sum_s \tilde{\lambda}_s \{g_s^{B,b}(\mathbf{q}) + \mathbb{E}^b[\psi_s^b(\mathbf{q}') \mid \mathbf{q}] - \psi_s^b(\mathbf{q})\}$ , each expectation under that regime's research policy. Subtracting the two regimes, the basic terms give  $\delta_s[h_s(r_s^1(\mathbf{q})) - h_s(r_s^0(\mathbf{q}))] \log q_s$ , the innovation channel, and the four adoption terms give the adoption channel; this is (23), exact term by term. For the invariant comparison, Theorem 4 applied to each regime gives  $g_C^b = (1 - \alpha) \sum_s \tilde{\lambda}_s \int g_s^{B,b} d\mathbb{P}^b$ ; subtracting gives  $g_C^1 - g_C^0 = (1 - \alpha) \sum_s \tilde{\lambda}_s (\int g_s^{B,1} d\mathbb{P}^1 - \int g_s^{B,0} d\mathbb{P}^0)$ , and adding and subtracting  $\sum_s \tilde{\lambda}_s \int g_s^{B,1} d\mathbb{P}^0$  splits the difference into the policy term, integrated against  $\mathbb{P}^0$ , and the distribution term,  $g_s^{B,1}$  integrated against  $\mathbb{P}^1 - \mathbb{P}^0$ ; this is (24). Only the research-policy term has a general sign:  $r_s^1 \geq r_s^0$  at every state makes it nonnegative; the adoption term and the gap-distribution term take either sign.  $\square$

## B Equilibrium Derivations and Auxiliary Results

### B.1 Prices, revenues, and the production-network block

Fix the licenses and hence  $\mathbf{A}_t$ . Define

$$\Omega_{ss'} \equiv \frac{\alpha m_{s's}}{\mu_s}, \quad \Psi \equiv (I - \Omega)^{-1}, \quad \tilde{\Omega} \equiv \text{diag}(\mu) \Omega, \quad \tilde{\Psi} \equiv (I - \tilde{\Omega})^{-1},$$

and

$$\boldsymbol{\lambda} = \Psi^\top \mathbf{b}, \quad \tilde{\boldsymbol{\lambda}} = \tilde{\Psi}^\top \mathbf{b}. \quad (35)$$

Row  $s$  of  $\Omega$  collects buyer  $s$ 's intermediate expenditures per dollar of its revenue, and  $\tilde{\Omega}_{ss'} = \alpha m_{s's}$  the corresponding production elasticities; the two coincide without markups. Thus  $\lambda_s$  is sector  $s$ 's revenue per unit of final expenditure and  $\tilde{\lambda}_s$  its cost-based Domar weight.

**Lemma B.1.** *Given  $\mathbf{A}_t$ , the static equilibrium exists and is unique, and:*

- (i) *labor receives  $\alpha \ell_s / \mu_s$  of sector  $s$ 's revenue, input  $s'$  receives  $\Omega_{ss'}$ , and operating profit before the license payment is  $(1 - \alpha / \mu_s) R_{st}$ ;*
- (ii)  *$R_{st} = \lambda_s C_t$ , with  $\boldsymbol{\lambda} \gg \mathbf{0}$  time invariant;*
- (iii)  *$\log C_t = \text{const} + (1 - \alpha) \tilde{\boldsymbol{\lambda}}^\top \log \mathbf{A}_t$ , with  $\tilde{\boldsymbol{\lambda}} \gg \mathbf{0}$ , so a one-percent increase in  $A_{st}$  raises consumption and every sector's revenue by  $(1 - \alpha) \tilde{\lambda}_s$  percent;*
- (iv) *a producer's optimized operating profit at productivity  $A$ , holding sector aggregates fixed, is proportional to  $A^{\varepsilon_s^\pi}$ , with  $\varepsilon_s^\pi = (1 - \alpha) / (\mu_s - \alpha) > 0$ .*

### B.2 Derivation of the production-network block

Fix a date and suppress time subscripts. Within sector  $s$ , the CES assembler gives each producer an iso-elastic demand, so with  $\mu_s > 1$  the Lerner rule  $p_{is} = \mu_s MC_s$  holds. The bargained intensity maximizes a strictly concave objective ( $\chi_s > 0$ ) that depends only on the own intensity and other sectors' aggregates ( $\theta_{ss} = 0$ ), so every producer chooses the same intensity; prices and quantities are then common and aggregate one for one to the sector. A single producer has measure zero, so a bargain's disagreement point leaves every aggregate unchanged.

*Proof of Lemma B.1.* (i) Cost minimization of the Cobb–Douglas bundle  $\Gamma$  gives  $w_s L_s = \ell_s E$  and  $P_{s'} M_{s's} = m_{s's} E$ , with  $E$  the minimized cost. Since  $Y = A^{1-\alpha} \Gamma^\alpha$ , variable cost is proportional to  $Y^{1/\alpha}$ , so  $MC_s = VC_s / (\alpha Y)$  and the markup rule gives  $VC_s = (\alpha / \mu_s) R_s$ ; the shares follow.

(ii) Market clearing for good  $s'$ , valued at  $P_{s'}$ , gives  $R_{s'} = b_{s'} C + \sum_s \Omega_{ss'} R_s$ , that is,  $\boldsymbol{\lambda} = \mathbf{b} + \Omega^\top \boldsymbol{\lambda}$ . The row sums of  $\Omega$  are  $\alpha(1 - \ell_s) / \mu_s < 1$ , so  $\rho(\Omega) < 1$ ,  $\Psi = \sum_n \Omega^n \geq 0$ , and  $\boldsymbol{\lambda} = \Psi^\top \mathbf{b} \gg \mathbf{0}$  because  $\mathbf{b} \gg \mathbf{0}$ .

(iii) With  $L_s = 1$  and the factor payments in (i), the log of the production function reads

$$(I - \tilde{\Omega}) \mathbf{p} = \text{diag}(\mathbf{g}) \log \mathbf{R} - (1 - \alpha) \log \mathbf{A} + \mathbf{v}, \quad \mathbf{g} \equiv (I - \tilde{\Omega}) \mathbf{1},$$

with  $v$  a constant vector. The row sums of  $\tilde{\Omega}$  are  $\alpha(1 - \ell_s) < 1$ , so  $\tilde{\Psi} \geq 0$  exists. Substituting  $\log \mathbf{R} = \log \lambda + \mathbf{1} \log C$  gives  $\mathbf{p} = \mathbf{p}^0 + \mathbf{1} \log C - (1 - \alpha)\tilde{\Psi} \log \mathbf{A}$ , with  $\mathbf{p}^0$  independent of  $\mathbf{A}$  and  $C$ . The numeraire  $\mathbf{b}^\top \mathbf{p} = 0$  and  $\mathbf{b}^\top \mathbf{1} = 1$  then pin down  $\log C = -\mathbf{b}^\top \mathbf{p}^0 + (1 - \alpha)\tilde{\lambda}^\top \log \mathbf{A}$ . Setting  $R_s = \lambda_s C$ ,  $Y_s = R_s / P_s$ ,  $c_s = b_s C / P_s$ , and inputs from (i) satisfies production, pricing, and market clearing; the household budget holds because labor income plus operating profit equals  $\sum_s R_s - \sum_{s,s'} \Omega_{ss'} R_s = C$ , and license payments are transfers. Each step has a unique solution, so the equilibrium is unique.

(iv) Taking the residual-demand shifter  $D_s$  and the bundle's unit cost  $c_s^\Gamma$  as given, the producer solves  $\max_Y \{D_s^{(\mu_s-1)/\mu_s} Y^{1/\mu_s} - c_s^\Gamma A^{-(1-\alpha)/\alpha} Y^{1/\alpha}\}$ . The first-order condition gives  $Y^* \propto A^{\mu_s(1-\alpha)/(\mu_s-\alpha)}$  and  $\pi^* \propto A^{\varepsilon_s^\pi}$ .  $\square$

**License fee and research cost.** By (iv), the operating-profit gain from frontier access is  $G_s = \pi_s^*(A_s^B)[(A_s/A_s^B)^{\varepsilon_s^\pi} - 1]$ . The Nash product  $[G_s - \phi]^{1-\zeta_s} \phi^{\zeta_s}$  is maximized at  $\phi = \zeta_s G_s$ , after which it is proportional to  $G_s$ , so the bargain chooses the intensity that maximizes the gain. At agreement,  $\pi_s^*(A_s) = (1 - \alpha/\mu_s)R_s$ , which gives (18). The household's first-order condition for research labor sets  $w_s^R = C$ ; with  $L_s^R = \lambda_s r_s$ , the research bill is  $R_s r_s$ .

### B.3 State space and transitions

This subsection records the state space and the bounds that the proofs of the results of Section 4 use. Research is unrestricted,  $r \in \mathbb{R}_+$ ; every bound below is implied by the innovator's objective.

**State space and transitions.** Let  $q_{s0} \in [1, \gamma_s]$ . Failure sends  $q_s$  to  $q_s^{\delta_s} \in [1, \gamma_s]$  and success to  $\gamma_s$ , so  $\mathcal{Q} = \prod_s [1, \gamma_s]$  is invariant and the state space  $\mathcal{Y} = \mathcal{Q} \times \{0, 1\}$  is compact. Innovation outcomes are independent across sectors, sector  $s'$  succeeding with probability  $h_{s'}(r_{s'}(\mathbf{q}, z))$ , and

$$q'_{s'} = \gamma_{s'} \text{ on success,} \quad q'_{s'} = q_{s'}^{\delta_{s'}} \text{ on failure,} \quad z' \sim \pi_{z'}. \quad (36)$$

Write  $a_s^*(\mathbf{q}, z)$  for the productivity ratio  $A_{st}/A_{st}^B$  of (7) at the profile  $\mathbf{x}(\mathbf{q}, z)$ , so that the fee (19) is  $f_s(\mathbf{q}, z) = \zeta_s(1 - \alpha/\mu_s)[1 - a_s^*(\mathbf{q}, z)^{-(1-\alpha)/(\mu_s-\alpha)}]$ . Since  $x_s = 0$  is feasible,  $a_s^*(\mathbf{q}, z) \geq 1$ , and the fee increases in the ratio toward  $\zeta_s(1 - \alpha/\mu_s)$ , so

$$0 \leq f_s(\mathbf{q}, z) \leq \bar{f}_s \equiv \zeta_s \left(1 - \frac{\alpha}{\mu_s}\right). \quad (37)$$

The rescaled value  $v_s$  of Lemma 3 therefore lies in  $[0, \bar{f}_s/(1 - \beta)]$ , so the gain from research over  $r = 0$  is at most  $\beta \bar{f}_s/(1 - \beta)$  while its cost is  $r$ ; every optimum satisfies

$$0 \leq r_s^*(\mathbf{q}, z) \leq \bar{r}_s^e \equiv \frac{\beta \bar{f}_s}{1 - \beta}. \quad (38)$$

#### B.4 Existence and structure of the research equilibrium

Fix a selection rule  $\mathbf{x}(\mathbf{q}, z)$ . The fee  $f_s(y) \in [0, \bar{f}_s]$  is then a function of the state  $y = (\mathbf{q}, z)$  alone, and the other innovators enter innovator  $s$ 's problem only through their success probabilities. Accordingly, measure a research policy by the success probability it delivers: a profile is  $\mathbf{h} = (h_s(y))_{s,y}$  with  $h_s(y) \in [0, \bar{h}_s]$ ,  $\bar{h}_s \equiv h_s(\bar{r}_s^e)$ , and  $\bar{r}_s^e$  the bound in (38). Let  $\mathcal{H} \equiv \prod_{s,y} [0, \bar{h}_s]$  with the pointwise order. Given  $\mathbf{h}_{-s}$ , next period's rival gaps are independent across sectors,  $q'_{s'} = \gamma_{s'}$  with probability  $h_{s'}(y)$  and  $q'_{s'} = q_{s'}^{\delta_{s'}}$  otherwise, so innovator  $s$ 's success premium is

$$\Delta_s(y; \mathbf{h}_{-s}) = \beta \sum_{z'} \pi_{zz'} \mathbb{E}_{\mathbf{h}_{-s}(y)} [v_s(\gamma_s, \mathbf{q}'_{-s}, z') - v_s(q_s^{\delta_s}, \mathbf{q}'_{-s}, z')], \quad (39)$$

where  $v_s$  is the value of Lemma 3 against  $\mathbf{h}_{-s}$ . Let  $\mathcal{B}_s(\mathbf{h}_{-s})(y) \equiv h_s(r_s^*(y))$  be the success probability of innovator  $s$ 's optimal policy against  $\mathbf{h}_{-s}$ , and  $\mathcal{B} = (\mathcal{B}_1, \dots, \mathcal{B}_S)$ . A Markov research equilibrium is a profile with  $\mathcal{B}(\mathbf{h}) = \mathbf{h}$ .

**Assumption B.1.** *For every sector  $s$  and state  $y$ , the success premium  $\Delta_s(y; \mathbf{h}_{-s})$  in (39) is nondecreasing in  $\mathbf{h}_{-s}$ .*

The assumption requires greater research by other innovators to raise the incremental continuation value of one's own success. The mechanism is the same complementarity that operates in adoption: rival successes widen the gaps into which their frontiers are sold and can strengthen the network support received after one's own success. The assumption is stated directly on the success premium because these effects propagate recursively through continuation values.

**Theorem B.1.** *Fix a selection rule. Under Assumption B.1:*

- (i) *A Markov research equilibrium exists. The set of equilibria is a nonempty complete lattice with least and greatest elements  $\underline{\mathbf{h}}$  and  $\bar{\mathbf{h}}$ , and every equilibrium  $\mathbf{h}$  satisfies*

$$\underline{\mathbf{h}} \leq \mathbf{h} \leq \bar{\mathbf{h}}.$$

- (ii) *Every equilibrium satisfies (38), and, conditional on the other innovators' equilibrium policies, each innovator's rescaled value is the unique bounded solution of (20).*

*Proof. Step 1:  $\mathcal{B}$  maps  $\mathcal{H}$  into itself.* Fix  $s$  and  $\mathbf{h}_{-s} \in \mathcal{H}$ . Research enters innovator  $s$ 's continuation only through  $h_s(r) \in [0, 1]$ , so the Bellman operator in (20) is a  $\beta$ -contraction on bounded functions of  $y$ , and  $v_s$  is its unique bounded fixed point. Since  $r = 0$  is feasible and  $0 \leq f_s \leq \bar{f}_s$ ,  $0 \leq v_s \leq \bar{f}_s / (1 - \beta)$ , so the success premium satisfies  $|\Delta_s| \leq \beta \bar{f}_s / (1 - \beta)$ . Hence the gain from research is bounded, while its cost is  $r$ , and no  $r > \bar{r}_s^e$  is optimal. For a given success premium  $\Delta_s$ , the research objective is  $-r + h_s(r)\Delta_s$ . It is strictly concave when  $\Delta_s > 0$  and strictly decreasing when  $\Delta_s \leq 0$ , so the maximizer  $r_s^*(y)$  is unique and lies in  $[0, \bar{r}_s^e]$ . Hence  $\mathcal{B}_s(\mathbf{h}_{-s})(y) \in [0, \bar{h}_s]$ .

*Step 2:  $\mathcal{B}$  is isotone.* Let  $\mathbf{h}_{-s} \leq \tilde{\mathbf{h}}_{-s}$ . By Assumption B.1,  $\Delta_s(y; \mathbf{h}_{-s}) \leq \Delta_s(y; \tilde{\mathbf{h}}_{-s})$  at every  $y$ . The objective  $-r + h_s(r)\Delta$  has increasing differences in  $(r, \Delta)$  because  $h_s$  is increasing, so by

Topkis's theorem the unique maximizer is nondecreasing in  $\Delta$ :  $r_s^*(y; \mathbf{h}_{-s}) \leq r_s^*(y; \tilde{\mathbf{h}}_{-s})$ . Since  $h_s$  is nondecreasing,  $\mathcal{B}_s(\mathbf{h}_{-s}) \leq \mathcal{B}_s(\tilde{\mathbf{h}}_{-s})$  pointwise. Thus  $\mathbf{h} \leq \tilde{\mathbf{h}}$  implies  $\mathcal{B}(\mathbf{h}) \leq \mathcal{B}(\tilde{\mathbf{h}})$ .

*Step 3: Tarski.*  $\mathcal{H}$  is a product of compact intervals with the product order, hence a complete lattice. By Steps 1 and 2,  $\mathcal{B}$  is an isotone self-map of  $\mathcal{H}$ , so by Tarski's fixed-point theorem its set of fixed points is a nonempty complete lattice; its least and greatest elements are  $\underline{\mathbf{h}}$  and  $\bar{\mathbf{h}}$ , and every fixed point lies between them. Fixed points of  $\mathcal{B}$  are exactly the Markov research equilibria, which gives (i). Part (ii) restates Step 1 at a fixed point.  $\square$

## B.5 Growth accounting

Write  $\psi_s(\mathbf{q}, z) \equiv \log a_s^*(\mathbf{q}, z)$  for the adoption component of productivity,  $I_{st} \in \{0, 1\}$  for the innovation outcome, and  $g_s^B(\mathbf{q}, z) \equiv [1 - \delta_s + \delta_s h_s(r_s^*(\mathbf{q}, z))] \log q_s$  for the conditional mean of basic-productivity growth; under the permanent regimes  $b \in \{0, 1\}$ , add the superscript  $b$ .

**Growth accounting.** With fixed shares the static allocation gives constant ratios  $M_{s't}/Y_{s't} = \Omega_{ss'}\lambda_s/\lambda_{s'}$  and  $c_{st}/Y_{st} = b_s/\lambda_s$ , so the production function reads  $\log Y_{st} = (1 - \alpha) \log A_{st} + \alpha \sum_{s'} m_{s's} \log Y_{s't}$  plus a constant. Differencing and stacking, with  $\tilde{\Omega}_{ss'} = \alpha m_{s's}$  and  $\rho(\tilde{\Omega}) \leq \max_s \alpha(1 - \ell_s) < 1$ ,

$$\Delta \log \mathbf{Y}_{t+1} = (1 - \alpha) \tilde{\Psi} \Delta \log \mathbf{A}_{t+1}, \quad \Delta \log \mathbf{C}_{t+1} = \mathbf{b}^\top \Delta \log \mathbf{Y}_{t+1} = (1 - \alpha) \tilde{\lambda}^\top \Delta \log \mathbf{A}_{t+1},$$

which are (40) and the Hulten identity, the second using  $\log C_t = \mathbf{b}^\top \log \mathbf{Y}_t + \text{const}$  and  $\tilde{\lambda}^\top = \mathbf{b}^\top \tilde{\Psi}$ . Basic productivity grows by  $\log q_{st}$  on success and by  $(1 - \delta_s) \log q_{st}$  on failure, so  $\Delta \log A_{s,t+1}^B = [1 - \delta_s + \delta_s I_{st}] \log q_{st}$ , and  $\log A_{st} = \log A_{st}^B + \psi_s(\mathbf{q}_t, z_t)$ . Substituting gives (21) realization by realization; taking the conditional expectation replaces  $I_{st}$  by  $h_s(r_s^*)$  and the basic term by  $g_s^B$ .

## B.6 Existence and uniqueness of the invariant law

Theorem 4 takes an invariant law of the state as given. The following lemma supplies it from the innovation process, without any continuity of the selected policies, which jump at the boundary of the region of multiple equilibria.

**Lemma B.2.** *Suppose research never dies out,*

$$\underline{p} \equiv \inf_{(\mathbf{q}, z)} \prod_{s=1}^S h_s(r_s^*(\mathbf{q}, z)) > 0,$$

*and either  $0 < \pi_{00}, \pi_{11} < 1$  or  $z_t \equiv b$  for some  $b \in \{0, 1\}$ . Then the selected Markov chain on  $\mathcal{Q} \times \{0, 1\}$  has a unique invariant law, and from every initial state its distribution converges to that law in total variation at a geometric rate. If instead no innovator researches at any state, gaps narrow deterministically and the point mass at  $\mathbf{q} = \mathbf{1}$  is the unique invariant law.*

*Proof.* At every state, all sectors succeed and  $z$  moves to a fixed state  $z^\circ$  with probability at least  $\varepsilon > 0$ : under  $0 < \pi_{00}, \pi_{11} < 1$ ,  $z^\circ = 1$  and  $\varepsilon = \underline{p} \underline{\pi}$  with  $\underline{\pi} = \min_{z, z'} \pi_{zz'}$ ; under  $z_t \equiv b$ ,  $z^\circ = b$  and  $\varepsilon = \underline{p}$ . The economy is then at  $(\gamma, z^\circ)$  whatever it was before. Hence the transition kernel satisfies  $P(y, \cdot) \geq \varepsilon \delta_{(\gamma, z^\circ)}$  for every state  $y$ : Doeblin's condition holds with the whole state space as a small set and minorizing measure  $\varepsilon \delta_{(\gamma, z^\circ)}$ , which gives a unique invariant law and convergence to it in total variation at rate  $(1 - \varepsilon)^t$  from any initial state (Stokey et al., 1989; Meyn and Tweedie, 2012).

If instead no sector researches at any state,  $q_{s,t+1} = q_{st}^{\delta_s}$  with probability one and  $q_{st} \rightarrow 1$ , so the point mass at  $\mathbf{q} = \mathbf{1}$  is the only invariant law of the gap process.  $\square$

## C Additional Theoretical Results

### C.1 Innovation: permanent regimes and negative returns

Section 4 states the persistence result and the closure result; this subsection records two companion results.

**Permanent regimes.** Consider permanent hype and its permanent absence. With  $\text{NAM}(\gamma) < 1$ , hype cannot matter; with negative standalone returns, the permanent absence of hype shuts research down.

**Corollary C.1.** *Under Assumption B.1, for every research equilibrium of each regime, including the greatest:*

- (i) *If  $\text{NAM}(\gamma) < 1$ , adoption is unique at every state, and permanent hype and its permanent absence induce identical fees and the same set of research equilibria, in particular the same greatest one.*
- (ii) *If  $\kappa_s < 0$  for every sector, the permanent absence of hype shuts down research,  $r_s^0 \equiv 0$ , so  $r_s^1(\mathbf{q}) \geq r_s^0(\mathbf{q})$  for every sector and state.*

**Negative standalone returns.** When every standalone return is negative, the comparison of Corollary C.1(ii) is immediate: no hype pays nothing.

**Corollary C.2.** *Suppose  $\kappa_s < 0$  for every sector. Then at every state the least adoption equilibrium is no adoption and the fee without hype is zero,  $f_s(\mathbf{q}, 0) = 0$  on  $\mathcal{Q}$ ; the fee difference  $f_s(\mathbf{q}, 1) - f_s(\mathbf{q}, 0)$  in the proof of Theorem 3 equals the fee under hype,  $f_s(\mathbf{q}, 1)$ , which is componentwise nondecreasing in the gap vector.*

#### Proofs.

*Proof of Corollary C.1.* (i) Every entry of  $\hat{\Theta}(\mathbf{q})$  is nondecreasing in  $\mathbf{q}$ , so  $\text{NAM}(\mathbf{q}) \leq \text{NAM}(\gamma) < 1$  on  $\mathcal{Q}$ , and Theorem 1 gives a unique adoption equilibrium at every state. Hence  $f_s(\mathbf{q}, 1) = f_s(\mathbf{q}, 0)$ , the two regimes define the same best-response map  $\mathcal{B}$ , and therefore the same set of fixed points.

(ii) Under the permanent absence of hype with  $\kappa_s < 0$ , the least adoption equilibrium is  $\mathbf{0}$  at every state and every fee is zero. Then  $v_s \equiv 0$  is the unique bounded solution of (20) whatever the other innovators do, with maximizer  $r_s = 0$ ; so  $r^0 \equiv \mathbf{0}$  is the unique research equilibrium, and  $r^1 \geq \mathbf{0} = r^0$ .  $\square$

*Proof of Corollary C.2.* At  $\mathbf{x} = \mathbf{0}$  the best response of  $s$  is  $[\hat{\kappa}_s]_{[0,1]} = 0$  because  $\kappa_s < 0$ , so  $\mathbf{0}$  is a fixed point and, being the smallest profile, the least equilibrium at every  $\mathbf{q}$ ; the pair's objective  $1 + x_s(q_s - 1)\kappa_s - \chi_s x_s^2/2$  is decreasing on  $[0, 1]$ , so  $x_s = 0$ ,  $a_s^*(\mathbf{q}, 0) = 1$ , and  $f_s(\mathbf{q}, 0) = 0$ .

At the greatest equilibrium the pair maximizes  $1 + x_s(q_s - 1)[\kappa_s + \sum_{s' \neq s} \theta_{ss'} \bar{x}_{s'}(\mathbf{q})] - \chi_s x_s^2/2$ , so the maximized ratio is  $a_s^*(\mathbf{q}, 1) = 1 + \mathcal{V}_s((q_s - 1)[\kappa_s + \sum_{s' \neq s} \theta_{ss'} \bar{x}_{s'}(\mathbf{q})])$  with  $\mathcal{V}_s$  the nondecreasing function (34). Since  $\bar{x}(\mathbf{q})$  is componentwise nondecreasing in  $\mathbf{q}$  (monotone comparative statics of extremal fixed points, (Topkis, 1998; Milgrom and Roberts, 1990)), so is the argument of  $\mathcal{V}_s$  wherever it is positive, and  $\mathcal{V}_s$  vanishes on the negative half-line; hence  $a_s^*(\mathbf{q}, 1)$  and, the license formula being increasing in the ratio,  $f_s(\mathbf{q}, 1)$  are nondecreasing in  $\mathbf{q}$ . With  $f_s(\mathbf{q}, 0) = 0$  the fee difference is  $f_s(\mathbf{q}, 1)$ .  $\square$

## C.2 Sectoral growth rates and wages

Fixed shares propagate productivity into sectoral output as well as into consumption:

$$\Delta \log \mathbf{Y}_{t+1} = (1 - \alpha) \tilde{\Psi} \Delta \log \mathbf{A}_{t+1}. \quad (40)$$

Sectoral real growth is generally heterogeneous. A common rate requires productivity growth to align with the input–output structure. Nominal expenditure shares nevertheless remain positive.

For an invariant law  $\mathbb{P}^*$  as in Theorem 4, write  $\mathbb{E}^*$  for its stationary mean. Mean sectoral quantity and price growth follow from the standard input–output mapping:

$$\mathbb{E}^*[\Delta \log \mathbf{Y}] = (1 - \alpha) \tilde{\Psi} \mathbb{E}^*[\Delta \log \mathbf{A}^B], \quad \mathbb{E}^*[\Delta \log \mathbf{P}] = g_C^* \mathbf{1} - \mathbb{E}^*[\Delta \log \mathbf{Y}]. \quad (41)$$

The sectors have a common mean quantity-growth rate  $g$  if and only if

$$(1 - \alpha) \mathbb{E}^*[\Delta \log \mathbf{A}^B] = (I - \tilde{\Omega})g\mathbf{1}, \quad \text{equivalently} \quad \mathbb{E}^*[\Delta \log A_s^B] = \frac{1 - \alpha + \alpha \ell_s}{1 - \alpha} g \quad \text{for every } s. \quad (42)$$

A pathwise balanced growth path with a common rate requires the analogous restriction each period. Constant productivity trends satisfying it, with constant adoption components, suffice.

Nominal shares remain positive regardless of real growth:  $R_{st}/\sum_{s'} R_{s't} = \lambda_s/\sum_{s'} \lambda_{s'} > 0$  and  $P_{st}c_{st}/C_t = b_s > 0$ . Under ergodicity, a slower-growing sector becomes asymptotically small in quantity relative to a faster-growing sector.

**Wage accounting.** Production labor is supplied inelastically at unit capacity, so the wage clears the market at the producers' common marginal revenue product: the factor-payment identity with  $L_{st} = 1$  and  $R_{st} = \lambda_s C_t$  gives

$$w_{st} = \frac{\alpha \ell_s}{\mu_s} \lambda_s C_t, \quad w_{st}^R = C_t. \quad (43)$$

Hence relative production wages, relative research-service wages, and  $w_{st}/w_{st}^R$  are independent of selection.

Coordination affects wage levels only through  $C_t$ .

### C.3 Received centrality and the left eigenvector

Fix  $\mathbf{q}$  and suppose that  $\widehat{\Theta}(\mathbf{q})$  is irreducible. Let  $\mathbf{v}, \mathbf{u} \gg \mathbf{0}$  be its right and left eigenvectors associated with  $\text{NAM}(\mathbf{q})$ , the right one proportional to  $\mathbf{c}$ , normalized so that  $\mathbf{u}^\top \mathbf{v} = 1$ . Sector  $s$  has received centrality  $v_s$ , and  $u_s$  is its weight in the left eigenvector. A sector has high received centrality when it is heavily involved in the pattern of mutually supporting adoption. A sector has a high weight  $u_s$  when adoption reaching it continues to circulate strongly through the network.

### C.4 Two-sector illustration: parameters

**Example C.1** (Two sectors). The economy drawn in Figures 2 and 3 has two symmetric sectors with  $\kappa_1 = \kappa_2 = -0.2$ ,  $\chi_1 = \chi_2 = 1$ ,  $\theta_{12} = \theta_{21} = \theta$ , and a common gap  $q$ . With two sectors  $\text{NAM}(\mathbf{q}) = \sqrt{\widehat{\theta}_{12}(q_1)\widehat{\theta}_{21}(q_2)} = (q-1)\theta/\chi$ , each best response is zero until the other sector's adoption exceeds  $-\kappa/\theta$  and has slope  $(q-1)\theta/\chi$  above it. The starting economy, panel (a) of both figures, has  $\theta = 1.3$  and  $q = 1.6$ , so  $\text{NAM} = 0.78$ . Panel (b) of Figure 2 has  $\theta = 2.6$  and  $q = 1.6$ , so  $\text{NAM} = 1.56$ ; panel (b) of Figure 3 has  $\theta = 1.3$  and  $q = 2.6$ , so  $\text{NAM} = 2.08$ . At  $\theta = 1.3$  the second equilibrium appears once both gaps reach  $1 + \chi/(\kappa + \theta) = 1 + 1/1.1 \approx 1.91$ . Figure 4 adds  $\gamma = q_0 = 2.6$ ,  $\delta = 0.98$ ,  $\alpha = 0.33$ ,  $\mu = 1.2$ ,  $\zeta = 0.5$ ,  $\beta = 0.96$ , and the research technology  $h_s(r) = \min\{1, \zeta\sqrt{r}\}$  with  $\zeta = 0.30$ ; it uses the two permanent regimes, as Proposition 1 does, and its path starts after forty-five periods without hype. Figure 5 uses the same parameters with the hype process  $\pi_{00} = 0.82$ ,  $\pi_{11} = 0.90$ .

### C.5 Within-sector complementarity

In this extension I allow for  $\theta_{ss} \geq 0$ . Producer  $i$ 's effective productivity is

$$\frac{A_{ist}}{A_{st}^B} = 1 + x_{ist}(q_{st} - 1) \left( \kappa_s + \theta_{ss}x_{st} + \sum_{s' \neq s} \theta_{ss'}x_{s't} \right) - \frac{\chi_s}{2}x_{ist}^2, \quad x_{st} = \int_0^1 x_{ist} di. \quad (44)$$

The optimal level of adoption and the license fee solve:

$$(x_{st}, \phi_{st}) \in \arg \max_{x \in [0,1], \phi \geq 0} \phi^{\zeta_s} \left[ \Pi_{st}^Y(A_{st}(x), \mathbf{P}_t, w_{st}) - \Pi_{st}^Y(A_{st}^B, \mathbf{P}_t, w_{st}) - \phi \right]^{1-\zeta_s}, \quad (45)$$

where  $A_{st}(x)$  is (44) at  $x_{ist} = x_{st} = x$ . While a producer takes the sector aggregate  $x_{st}$  as given, the innovator licenses to every producer in the sector. By doing so, it anticipates that the higher will be the level of adoption within the sector, and the higher will be the license it gets. Thus, the optimal  $x$  solves:

$$\max_{x \in [0,1]} x(q_{st} - 1) \left( \kappa_s + \theta_{ss}x + \sum_{s' \neq s} \theta_{ss'}x_{s't} \right) - \frac{\chi_s}{2}x^2. \quad (46)$$

**Assumption C.1.**  $2(\gamma_s - 1)\theta_{ss} < \chi_s$  for every  $s$ .

**Lemma C.1.** Under Assumption C.1, for every  $\mathbf{q} \in \mathcal{Q}$  the objective (46) is strictly concave, and the sector's best response to  $\mathbf{x}_{-s}$  is

$$x_s = \max \left\{ \min \left\{ \frac{(q_s - 1)(\kappa_s + \sum_{s' \neq s} \theta_{ss'} x_{s'})}{\chi_s - 2(q_s - 1)\theta_{ss}}, 1 \right\}, 0 \right\} \quad (47)$$

which is (16) with  $\chi_s$  replaced by  $\chi_s - 2(q_s - 1)\theta_{ss} > 0$ .

The statement trivially follows by taking the first order condition. Under the assumption C.1 the objective is strictly concave, therefore the best response is unique.

An adoption equilibrium is a fixed point of (47) across sectors, which is the best-response map of Section 3 with  $\hat{\Theta}(\mathbf{q})$  given the diagonal entries  $\hat{\theta}_{ss}(q_s)$ . Every argument in Sections 3–5 and Appendix A uses that map through three properties, and each holds with the diagonal: the map is nondecreasing in  $\mathbf{x}$ ; its coefficients vanish at  $q_s = 1$ , increase in  $q_s$ , and row  $s$  scales with  $q_s - 1$ ; and the fee is (18) at a ratio that is at least one and nondecreasing in the others' adoption.

It might be useful to discuss the relevance of the assumption above. Since  $\hat{\theta}_{ss} < 1$ , the source of multiplicity is not the within sector complementarity. In particular, without the assumption, the objective function (46) is convex for  $q_s$  near  $\gamma_s$ . The best response is then a step, still monotone, so Lemma 1 survives but the affine structure the remaining results use does not.

## D Estimation Appendix

### D.1 Adoption intensity and the directed network

For firm  $i$ , sector  $s$ , technology  $k$ , and year  $t$ , the raw data contain technology-related postings and total postings. Using the posting share adoption is:

$$x_{iskt} = \min \left\{ \frac{\text{posting firm } i \text{ mentioning technology } k \text{ in sector } s \text{ at } t}{\text{total posting firm } i \text{ in sector } s \text{ at } t}, \text{reference}_k \right\}.$$

where  $\text{reference}_k$  is the 95th percentile of the technology's shares across sectors and years. Five technologies have no positive share in those years, leaving 382 technologies. I use the firm level data for the construction of the instrument, but then I aggregate the adoption at sector level throughout the analysis, as specified in the main body of the paper.

Furthermore, I restrict the graph to the 25 largest in each row across 278 sectors which leave 6,950 links. As for the links outside the 25 largest in each row, I record them as  $G^{\text{tail}}$ .

**Details of Example in Figure 6.** Technology  $b$  is used by all three sectors, so  $\text{idf}_b = \log(3/3) = 0$ . Technologies  $a$  and  $c$  are each used by two sectors, so  $\text{idf}_a = \text{idf}_c = \log(3/2) = 0.405$ . Sector 1 places 0.8 of its portfolio on  $a$  and 0.2 on  $b$ ,  $w_{1a} = 0.8$  and  $w_{1b} = 0.2$ , and sector 2 uses both,  $e_{2a} = e_{2b} = 1$ , so  $G_{12} = 0.8 \times 0.405 + 0.2 \times 0 \approx 0.32$ . Sector 3 uses  $b$  and  $c$ ,  $e_{3a} = 0$ , so  $G_{13} = 0.2 \times 0 = 0$ . The links into sectors 2 and 3 follow in the same way. Sector 2 places 0.2 on  $a$ , so  $G_{21} = 0.2 \times 0.405 \approx 0.08$ . Through technology  $c$ ,  $G_{23} = 0.4 \times 0.405 \approx 0.16$  and  $G_{32} = 0.5 \times 0.405 \approx 0.20$ , and  $G_{31} = 0$ , since sectors 1 and 3 share technology  $b$  alone.

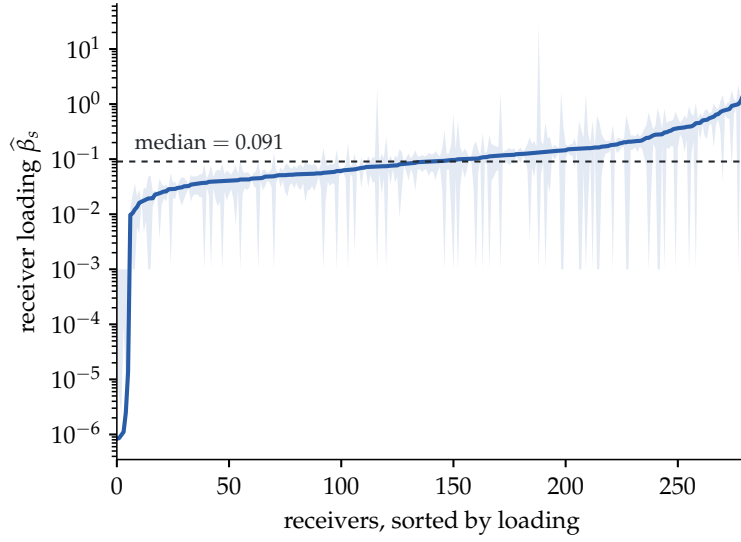
### D.2 Annual control-function Tobit

I keep a receiver if two conditions are satisfied. First, the first-stage is strong enough,  $F \geq 10$ . Second, it has enough interior observations to pin down a slope, *i.e.*, at least 15 of its 3,096 strictly between zero and one. Of the 305 receivers, 294 pass the first condition and 282 both. Their median loading is 0.091 and their median first-stage  $F$  is 68. If I leave the slope unrestricted, it is still positive for 276 of the 282. In the 278-sector quantitative sample, 277 receivers have in-degree complementarity and only NAICS 4859 fails the screen; the median loading is 0.090 and the median  $F$  is 68. All 277 intercepts are negative.

The loadings that enter the NAM and the quantitative model are empirical-Bayes adjusted. I replace each fitted loading with a precision-weighted average of itself and the median fitted loading, 0.090, where the weights are the inverse of its clustered variance and the inverse of the dispersion of the fitted loadings beyond their sampling variance. The median stays where it is and half of the loadings move by less than one percent.

The NAM on the 278 sectors is 1.179 with the empirical-Bayes loadings and 1.180 with the fitted loadings. It is above one in every bootstrap replication, and it is 1.15 on the 263 receivers

**Figure D.1: RECEIVER LOADINGS**



*Notes:* The bands are two-standard-error on a log scale.

with a first-stage  $F$  of at least 30. I bootstrap by resampling technologies, re-estimating both stages, reapplying the screen, and recomputing the empirical-Bayes loadings.

### D.3 Input-output and knowledge-network robustness

This section compares the estimated effective network with the input-output (IO) and knowledge networks, and checks that the response of adoption holds once adoption through those networks is controlled for.

IO exposure can happen in two directions: adoption by a sector's customers can raise demand for its output; and adoption by its suppliers can lower the cost of its inputs. Let  $F_{j \rightarrow s}$  be the 2007 BEA delivery from seller  $j$  to buyer  $s$ . With receivers in rows, sources in columns, and zero diagonals, the two networks are

$$W_{sj}^D = \frac{F_{s \rightarrow j}}{\sum_{r \in S, r \neq s} F_{s \rightarrow r}}, \quad W_{sj}^U = \frac{F_{j \rightarrow s}}{\sum_{r \in S, r \neq s} F_{r \rightarrow s}}, \quad (48)$$

where  $S$  is the common set of sectors. The knowledge network  $W^K$  is the row-normalized matrix of patent citations between sectors up to 2006.

Table D.1 reports the correlations between the entries of the estimated effective network and those of the three other networks, across all directed pairs of these 203 sectors, for both the baseline and the strict pre-period network. Figure 7 in the main text plots the networks on the same sectors.

As a placebo, I shuffle sector labels within each technology and year, rebuild the co-adoption network from the shuffled data, and re-run a pooled IV regression of adoption on its network

**Table D.1:** Entrywise correlations between networks

|                                                             | Estimated effective network $\Theta^A$ |                   |
|-------------------------------------------------------------|----------------------------------------|-------------------|
|                                                             | Full panel                             | Strict pre-period |
| <i>Panel A. Estimated effective network paired with</i>     |                                        |                   |
| Demand IO $W^D$                                             | 0.029                                  | 0.033             |
| Input IO $W^U$                                              | 0.091                                  | 0.095             |
| Knowledge $W^K$                                             | 0.088                                  | 0.092             |
| <i>Panel B. Alternative networks paired with each other</i> |                                        |                   |
| $W^D$ and $W^U$                                             |                                        | 0.048             |
| $W^D$ and $W^K$                                             |                                        | 0.077             |
| $W^U$ and $W^K$                                             |                                        | 0.094             |

*Notes:* Pearson correlations across the 41,006 directed off-diagonal pairs of the 203 common sectors, including zeros. The IO and knowledge networks are row-normalized after restriction to these sectors. The full-panel network uses the loadings and top-25 graph of Section 6; the strict pre-period network refits the loadings with technology portfolios and reference shares measured in 2007, 2010, and 2011 alone (382 technologies).

lag. The shuffle breaks the pattern of who adopts with whom. A second comparison uses a random network with the same link density, rescaled to the observed NAM. Table D.2 reports the coefficients.

**Table D.2:** Directed-network permutation placebo

| Adjacency                 | Radius | IV loading | Amplification |
|---------------------------|--------|------------|---------------|
| Observed directed network | 1.336  | 0.567      | 0.758         |
| Permuted sector labels    | 0.997  | −0.040     | −0.040        |
| Random network            | 1.336  | 0.369      | 0.493         |

*Notes:* Pooled instrumental-variable loading of residualized adoption on its network lag and amplification, the loading times the spectral radius. One permutation draw and one random-network draw.

## D.4 Alternative graphs

Alternative graphs G use four-digit NAICS sectors and data through 2006. The equipment graphs use correlations in Compustat machinery and equipment growth after controlling for sales growth, firm size, equipment intensity, acquisition disclosure, and firm and year effects. The two additional equipment specifications scale changes by lagged assets and restrict acquisitions to one percent of lagged assets. Common-customer graphs use similarity in sales across named major customers and the share of a receiver's sales to customers shared with a source. Market-return graphs use CRSP sector returns after removing the Fama–French three factors and momentum. They use residual correlations, partial correlations, and lagged return prediction. Occupational graphs use similarity in employment shares from the BLS May 2006 industry–occupation data, with equal weights or greater weights on occupations with smaller summed employment shares

across sectors. Patent graphs use similarity in patent classifications and citations from receiver sectors to source sectors. Input-adjusted sales graphs use correlations and lagged prediction in Compustat sales growth after controlling for capital, cost of goods sold, employment, and firm and year effects. Patent-announcement graphs use cross-sector responses of three-day abnormal stock returns to patent values and patent counts. Patent grant dates and values come from [Kogan et al. \(2017\)](#). For each graph, I re-estimate both stages of the control-function Tobit and compare the multiplier with the original graph, the graph of Section 6.2, on the same sources and common supported receivers (Table D.3). Restricting the original graph to the sectors an alternative covers lowers its multiplier, since the spectral radius of a principal submatrix of a nonnegative matrix is at most that of the full matrix. Seventeen alternatives yield multipliers above one and above the matched original graph. The directed return-prediction graph is acyclic and yields a multiplier of zero. The original graph combines wider sector coverage with the smallest multiplier in each of the seventeen comparisons, so the quantitative results of Section 7 use the most conservative network. Shared demand, competition, and common shocks also contribute to these alternative links; the causal interpretation of the adoption response rests on the granular instrument.

**Table D.3:** Alternative graphs and adoption responses

| Alternative graph                   | Four-digit sectors | Supported receivers | $\widehat{\text{NAM}}$ |                   | Median loading | Median first-stage $F$ |
|-------------------------------------|--------------------|---------------------|------------------------|-------------------|----------------|------------------------|
|                                     |                    |                     | Original graph         | Alternative graph |                |                        |
| <i>Equipment</i>                    |                    |                     |                        |                   |                |                        |
| Equipment growth                    | 109                | 104                 | 1.120                  | 2.016             | 0.122          | 59.6                   |
| Equipment change / lagged assets    | 109                | 104                 | 1.115                  | 2.012             | 0.130          | 65.1                   |
| Equipment growth, low acquisitions  | 86                 | 80                  | 1.093                  | 1.601             | 0.122          | 57.0                   |
| <i>Common customers</i>             |                    |                     |                        |                   |                |                        |
| Sales-weighted similarity           | 130                | 111                 | 1.124                  | 1.781             | 0.105          | 43.3                   |
| Directed sales exposure             | 130                | 118                 | 1.027                  | 1.762             | 0.122          | 68.4                   |
| <i>Market returns</i>               |                    |                     |                        |                   |                |                        |
| Value-weighted returns              | 158                | 140                 | 1.025                  | 1.838             | 0.124          | 53.9                   |
| Equal-weighted returns              | 158                | 140                 | 1.117                  | 1.642             | 0.117          | 44.9                   |
| Value-weighted returns, three firms | 193                | 174                 | 1.051                  | 1.978             | 0.132          | 52.6                   |
| Partial correlations                | 134                | 115                 | 1.008                  | 1.560             | 0.119          | 57.7                   |
| Directed return prediction          | 134                | 8                   | 0.000                  | 0.000             | 0.096          | 58.4                   |
| <i>Occupations</i>                  |                    |                     |                        |                   |                |                        |
| Employment-share similarity         | 213                | 137                 | 0.980                  | 4.951             | 0.109          | 30.8                   |
| Rare-occupation weights             | 213                | 140                 | 0.979                  | 4.920             | 0.103          | 28.2                   |
| <i>Patents</i>                      |                    |                     |                        |                   |                |                        |
| Patent-portfolio similarity         | 119                | 110                 | 0.935                  | 2.309             | 0.173          | 60.1                   |
| Directed patent citations           | 119                | 110                 | 1.112                  | 1.178             | 0.092          | 79.9                   |
| <i>Input-adjusted sales</i>         |                    |                     |                        |                   |                |                        |
| Residual sales correlation          | 156                | 145                 | 1.116                  | 2.227             | 0.144          | 51.0                   |
| Directed sales prediction           | 147                | 146                 | 1.118                  | 1.957             | 0.135          | 70.5                   |

Continued on next page

Table D.3 (continued)

| Alternative graph                  | Four-digit sectors | Supported receivers | $\widehat{\text{NAM}}$ |                   | Median loading | Median first-stage $F$ |
|------------------------------------|--------------------|---------------------|------------------------|-------------------|----------------|------------------------|
|                                    |                    |                     | Original graph         | Alternative graph |                |                        |
| <i>Patent-announcement returns</i> |                    |                     |                        |                   |                |                        |
| Patent values                      | 100                | 93                  | 0.998                  | 1.661             | 0.155          | 99.9                   |
| Patent counts                      | 100                | 97                  | 1.026                  | 1.363             | 0.114          | 81.2                   |

*Notes:* Four-digit sectors is the sector universe each alternative covers. Both stages of the control-function Tobit are re-estimated for each graph on annual adoption over 2012–2019. The original graph is restricted to the same sectors; each alternative keeps up to 25 source links per receiver, scaled to the original row sums.  $\widehat{\text{NAM}}$  is the spectral radius of  $\text{diag}(\widehat{\beta})G$ .

## E Quantification Appendix

This section is organized as follows. First, it records the empirical-to-structural mapping, then extends the model to the finite set of technologies observed in the data. The remaining subsections describe details on the numerical method and counterfactuals.

### E.1 Empirical objects and structural mapping

The quantitative model is built on the estimates of Section 6, on the same  $S = 278$  four-digit NAICS sectors and  $K = 387$  technologies. The graph becomes the primitive network,  $\Theta = G$ . The receiver loadings  $\hat{\beta}$ , set how strongly each sector responds to support. Mean adoption by sector pins down the standalone returns.

Let  $\bar{x}_s$  be sector  $s$ 's mean adoption across technologies over 2012–2019, measured as in the Tobit. Evaluating the interior best response at sector means,  $\bar{x}_s \simeq \hat{\beta}_s(\kappa_s + \sum_{s'} G_{ss'} \bar{x}_{s'})$ , gives

$$\kappa_s = \frac{\bar{x}_s}{\hat{\beta}_s} - \sum_{s'} G_{ss'} \bar{x}_{s'} \quad (49)$$

for the 277 supported receivers.<sup>17</sup> I keep these returns fixed in the dynamic calibration and in all counterfactuals.

The standalone return is negative in 261 of the 278 sectors (Figure E.1), with a median of  $-1.13$  and tenth and ninetieth percentiles of  $-1.68$  and  $-0.31$ . The 17 sectors with positive returns have a high level of adoption but they respond little to changes in other sectors' adoption: their median loading is at the seventh percentile and their median adoption at the eighty-third. They include scientific research and development, architecture and engineering, computer systems design, universities, schools, pharmaceuticals, measuring instruments, hospitals, physicians, outpatient care, electric power, and public administration; the first four are among the largest sources in the estimated effective network.

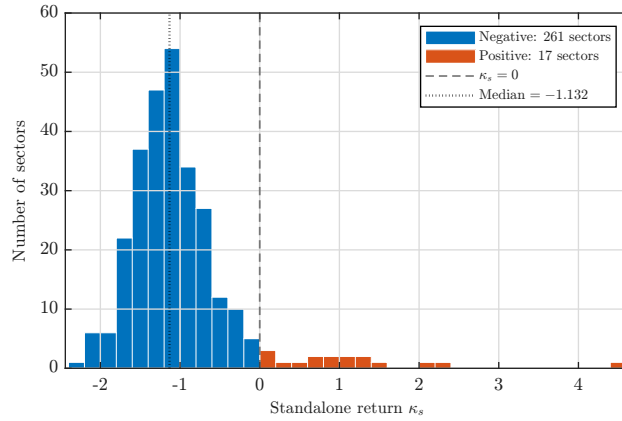
### E.2 Hype measurement

Hype is measured from text analysis of the 10-K filings. The sample is the firms in a fixed S&P 500 roster and their EDGAR 10-K filings from 1996 to 2023. In Item 7, management's discussion and analysis, I count a predetermined set of 23 adoption, implementation, and technology synonyms. With  $H_f$  hits and  $W_f$  words in the Item 7 text of filing  $f$ , firm  $i$ 's intensity in year  $t$  is  $L_{it} = 1000 \sum_{f \in \mathcal{F}_{it}} H_f / \sum_{f \in \mathcal{F}_{it}} W_f$ .

Consistent with the model Markov process for hype, I estimate a Markov-switching model of the standardized index  $y_t$  (Hamilton, 1989). In state  $j$ ,  $y_t \sim \mathcal{N}(\mu_j, \sigma_j^2)$ , with  $\mu_1 > \mu_0$  so that hype is the state with more adoption language, and the state follows a Markov chain with transition probabilities  $\pi_{ij} = \Pr(z_t = j \mid z_{t-1} = i)$ . The Hamilton filter gives the likelihood, and the

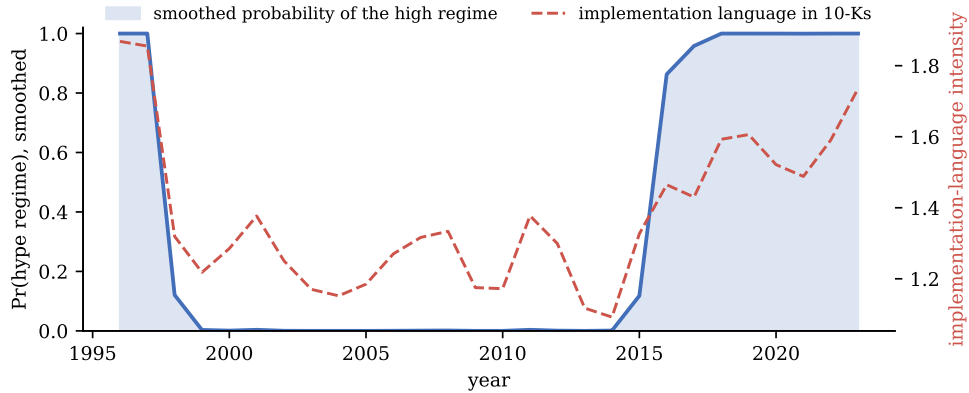
<sup>17</sup>For NAICS 4859, whose Tobit fit fails the support screen,  $\kappa_s$  is set to  $-0.0093$ .

**Figure E.1: DISTRIBUTION OF STANDALONE RETURNS ACROSS THE 278 SECTORS**



Notes: Standalone returns are from (49). The vertical lines mark zero and the median.

**Figure E.2: 10-K LANGUAGE INTENSITY AND THE SMOOTHED HYPE PROBABILITY**



Notes: The dashed line is the annual mean of firm-level keyword hits per 1,000 Item 7 words; the shaded curve is  $\Pr(z_t = 1 \mid y_{1996:2023})$ , obtained by backward smoothing after Hamilton filtering.

smoothed probability of hype,  $p_{t|T}(1)$ , plotted in Figure E.2. The estimates are  $\hat{\pi}_{00} = 0.920$  and  $\hat{\pi}_{11} = 0.935$ , with standard errors 0.061 and 0.073.

As a robustness I run different specification to back out a target for the hype process. Table E.1 summarizes them. Adding firm excess return volatility to the language series, or letting the transition probabilities depend on the lagged index (Kim and Yoo, 1995; Filardo, 1994), keeps both probabilities between 0.85 and 0.94. Michigan consumer expectations (University of Michigan Surveys of Consumers, 2026) give (0.854, 0.888) in a Gaussian HMM and (0.738, 0.839) in an MS-AR(1). Two principal components of four nontext series give values between 0.68 and 0.92.

**Table E.1: State persistence in independent indicators and filing-language specifications**

| Measurement and specification                 | Sample    | $\hat{\pi}_{00}$ | $\hat{\pi}_{11}$ |
|-----------------------------------------------|-----------|------------------|------------------|
| <i>Independent indicators: no filing text</i> |           |                  |                  |
| Michigan consumer expectations, Gaussian HMM  | 1961–2023 | 0.854            | 0.888            |

Continued on next page

**Table E.1:** State persistence in independent indicators and filing-language specifications (continued)

| Measurement and specification                                  | Sample    | $\hat{\pi}_{00}$ | $\hat{\pi}_{11}$ |
|----------------------------------------------------------------|-----------|------------------|------------------|
| Michigan consumer expectations, MS-AR(1)                       | 1961–2023 | 0.738            | 0.839            |
| Text-free PCA, macro component, MS-AR(1)                       | 1965–2023 | 0.750            | 0.906            |
| Text-free PCA, sentiment component, MS-AR(1)                   | 1965–2023 | 0.917            | 0.680            |
| <i>Filing-language specifications: measurement sensitivity</i> |           |                  |                  |
| 10-K language, switching-mean/variance model                   | 1996–2023 | 0.920            | 0.935            |
| 10-K language + firm excess volatility, Gaussian HMM           | 1996–2023 | 0.923            | 0.930            |
| 10-K + firm excess volatility, lagged-index TVTP               | 1996–2023 | 0.931            | 0.941            |
| 10-K + firm excess volatility, Gaussian HMM                    | 1996–2019 | 0.920            | 0.852            |
| 10-K + firm excess volatility, lagged-index TVTP               | 1996–2019 | 0.933            | 0.906            |

*Notes:* The principal components use (Baker and Wurgler, 2006), JLN uncertainty (Jurado et al., 2015), Michigan expectations, and market-return volatility. Firm excess volatility is market-adjusted monthly return volatility relative to the firm's recent history. CRSP volatility ends in 2019; over 2020–2023 the full-sample specifications use language alone.

### E.3 Calibration details

I internally calibrate jointly  $\zeta$ ,  $\chi$ ,  $\gamma$ ,  $\pi_{00}$ , and  $\pi_{11}$ . The standalone returns from (49), diffusion parameter  $\delta$ , input-output matrix, expenditure shares, and  $\Theta = G$  remain externally calibrated. For the moment identifying hype, I use the transition probabilities as explained in Section E.2. Below I describe in detail the remaining targets/sources.

**Input-output objects.** I retrieve the production network from the 2015 producer-price Use table before redefinitions, with one commodity per industry. Its 66 private accounts, with government and real estate consolidated, are split proportionally across the 278 sectors, and the entries with negative added values in intermediate and final use are set to 0. With rows as buyers, let  $U_{ss'}$  be sector  $s$ 's purchases from  $s'$ ,  $GO_s$  its gross output, and  $W_s L_s$  its compensation. The input shares are  $\Omega_{ss'} = U_{ss'}/GO_s$  and the cost share of gross output is  $v_s = W_s L_s / GO_s + \sum_{s'} \Omega_{ss'}$ . The markup is  $\mu_s = \alpha / v_s$ , which exceeds one when  $\alpha > \max_s v_s$ , and the labor and input shares of cost are  $\ell_s = W_s L_s / (v_s GO_s)$  and  $m_{s's} = \Omega_{ss'} / v_s$ , which sum to one.

**Diffusion.** I estimate  $\delta$  from the persistence of firms' productivity gaps to their sector's frontier in Compustat. For firm  $i$  in sector  $s$  and year  $t$ , revenue TFP is

$$\log \text{tfp}_{it} = \log \text{sales}_{it} - \text{ls}_s \log \text{emp}_{it} - (1 - \text{ls}_s) \log \text{ppe}_{it},$$

where  $\text{emp}$  is employment,  $\text{ppe}$  gross plant, and  $\text{ls}_s$  the sector's labor share, the median ratio of staff expense to sales over 2007 and 2010–2019, bounded between 0.10 and 0.90. Revenue TFP is winsorized at the first and ninety-ninth percentiles within each sector and year. The sector's frontier  $\overline{\text{tfp}}_{st}$  is the ninetieth percentile of  $\log \text{tfp}_{it}$  in the sector and year, and the firm's gap is

$$\text{gap}_{it} = \max\{\overline{\text{tfp}}_{st} - \log \text{tfp}_{it}, 0\}.$$

The diffusion parameter is the slope of the pooled regression

$$\text{gap}_{i,t+1} = c + \delta \text{gap}_{it} + \varepsilon_{i,t+1}.$$

**Arrival probabilities.** Breakthroughs are the patents in the top decile of the importance measure of Kelly et al. (2021). The successor of a breakthrough is the first later breakthrough that cites it, in any sector, and the waiting time is the number of years between their issue dates, with successors issued in the same year counted in the first year. Patents are classified by technology in the Cooperative Patent Classification (CPC), so I assign them to NAICS-4 sectors with the concordance of Goldschlag et al. (2020). Sector  $s$ 's arrival probability at horizon  $h$  is the weighted share of its breakthroughs that have a successor within  $h$  years,

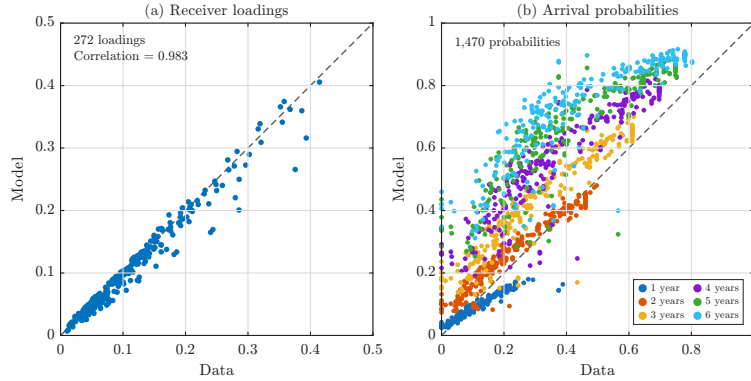
$$F_s(h) = \frac{\sum_n w_{ns} \mathbf{1}\{T_n \leq h\}}{\sum_n w_{ns}}, \quad h = 1, \dots, 6,$$

where  $T_n$  is the waiting time of breakthrough  $n$ . Every cohort of breakthroughs is observed for at least six years.

#### E.4 Fit and robustness

Figure E.3 reports data vs model for the coefficient  $\hat{\beta}_s$  and arrival probabilities at all six horizons.

**Figure E.3: MODEL FIT: RECEIVER LOADINGS AND ARRIVAL PROBABILITIES**



*Notes:* Panel (a) plots empirical and model receiver loadings. Panel (b) plots first-successor arrival probabilities at horizons of one through six years for 245 sectors. Colors identify the horizons. Each dashed line is the 45-degree line.

#### E.5 Policy computation

The ranking of Table 5 averages over the 12,416 simulated technology-periods, under the estimated hype process and under permanent hype, at which the least and greatest equilibria differ. The outward centrality in (33) is

$$c_s^{\text{out}}(\mathbf{q}) = \sum_{n \geq 0} \sum_{j \in \mathcal{A}: \bar{x}_j < \bar{x}_j} \lambda_j [\hat{\Theta}(\mathbf{q})^n]_{js}, \quad (50)$$

where  $\mathcal{A}$  is the set of sectors with interior adoption at the low adoption equilibrium. On  $\mathcal{A}$  the responses are  $\partial \underline{x}_j / \partial \tau_s^A = \frac{q_s - 1}{\chi_s} \sum_{n \geq 0} [\widehat{\Theta}(\mathbf{q})^n]_{js}$ . After the restriction to  $\mathcal{A}$ , the spectral radius is at most 0.992 at every observation, hence the series converges. At the common wedge of 0.431, which closes 99 percent of the gap,  $\nu = 11.47$ , and each of the 268 excluded sectors satisfies (33) with a strict inequality.<sup>18</sup> The wedge changes the private return to adoption. For task  $(s, k)$ , productivity before subsidy is

$$a_{skt}^0 = 1 + x_{skt}(q_{skt} - 1) \left( \kappa_s + \sum_{s' \neq s} \theta_{ss'} x_{s'kt} \right) - \frac{\chi_s}{2} x_{skt}^2, \quad (51)$$

and the producer–innovator pair bargains over the operating-profit gain evaluated at  $a_{skt}^\tau = a_{skt}^0 + x_{skt}(q_{skt} - 1)\tau_s^A$ . With  $G_s(a) = K(1 - \alpha/\mu_s)(1 - a^{-\varepsilon_s^\pi/K})$  the gain per unit of task revenue, the government pays the pair

$$\mathcal{T}_{skt}^I = \frac{C_t \lambda_s}{K} [G_s(a_{skt}^\tau) - G_s(a_{skt}^0)], \quad (52)$$

financed by a lump-sum tax. The pair adopts as if its standalone return were  $\kappa_s + \tau_s^A$ , while output uses  $a_{skt}^0$ . The targeted subsidy sets  $\tau_s^A = 0.323$  in the ten sectors of Table 5 and zero elsewhere.

An R&D subsidy rate  $\tau^R \in [0, 1)$  lowers the innovator's research cost from  $C_t L_{st}^R$  to  $(1 - \tau^R)C_t L_{st}^R$ . The financing is through a lump-sum.

With  $\mathcal{T}_t$  the total subsidy payments in period  $t$ , the fiscal requirement is  $100(1 - \beta) \mathbb{E}_0 \sum_{t \geq 0} \beta^t \mathcal{T}_t / C_t$ , computed on the paths of the welfare. Table 6 reports it relative to the research bill of the estimated hype process.

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<sup>18</sup>A common wedge makes the high adoption equilibrium arise at an infinite amount for some sectors. Hence, for numerical reason, I impose that the subsidy closes 99 percent of the gap.

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